LECTURE NOTES

ON

ELECTRONIC MEASUREMENTS AND INSTRUMENTATION

Branch: ECE **Year & Semester: IV & 1**

Syllabus

Unit – I

Block Schematic Of Measuring Systems

Performance Characteristics, Static Characteristics, Accuracy, Precision, Resolution, Types Of Errors, Gaussian Error ,Root Sum Squares Formulae, Dynamic Characteristics- Repeatability, Reproducibility, Fidelity, Lag; Measuring Instruments: DC Voltmeters, D-Arsonval Movement,DC Current Meters , AC Voltmeters and Current Meters, Ohmmeters, Multimeter, Meter Protection, Extension Of Range, True Rms Responding Voltmeters, Specification Of Instruments.

Unit-II

Signal analyzers:

AF,HF Wave Analyzers, Harmonic Distortion, Heterodyne Wave Analyzers, Spectrum Analyzers, Power Analyzers, Capacitance –Voltage Meters, Oscillators

Signal generators:

AF,HF Signal Generators, Sweep Frequency Generators, Arbitrary Waveform Generator, Video Signal Generators And Specifications

Unit – III

Oscilloscopes:

CRT, Block Schematic Of CRO, Time Base Circuits, Lissajous Figures, CRO Probes, High Frequency CRO Considerations, Delay Lines,

Applications:
Measurement Of Time, Period And Frequency Specifications

Special purpose oscilloscopes:

Dual trace, Dual beam CROs, sampling oscilloscope, storage oscilloscope, digital storage oscilloscope,

UNIT IV

TRANSDUCERS:

Classification, Strain Gauges, Bounded, Un-Bounded; Force And Displacement Transducers, Resistance Thermometers, Hotwire Anemometers, LVDT, Thermocouples, Synchronous, Special Resistance thermometers, Digital Temperature Sensing System, Piezoelectric Transducers, Variable Capacitance Transducers, Magneto Strictive Transducers

UNIT V

BRIDGES:

Wheat Stone Bridge, Kelvin Bridge, Maxwell’s Bridge.

MEASUREMENT OF PHYSICAL PARAMETERS:

Flow measurement, displacement meters, liquid level measurement, measurement of humidity and moisture, velocity, force, pressure – high pressure, vacuum level, temperature- measurements, data acquisition systems

IX. List of Text Books / References / Websites / Journals / Others

Text Books:

1. Electronic Instrumentation, - H.S.Kalsi, 2nd Edition, Tata McGraw Hill, 2004.

2. Modern Electronic Instrumentation and Measurement TechniquesA.D.Helfrick and W.D. Cooper, 5th Edition, PHI, 2002.

Reference Books:

1. Transducers and display systems - B.S.Sonde.

2. Electronic measurements and Instrumentation – B. M. Oliver and J.M. Cage, TMH, 2009.

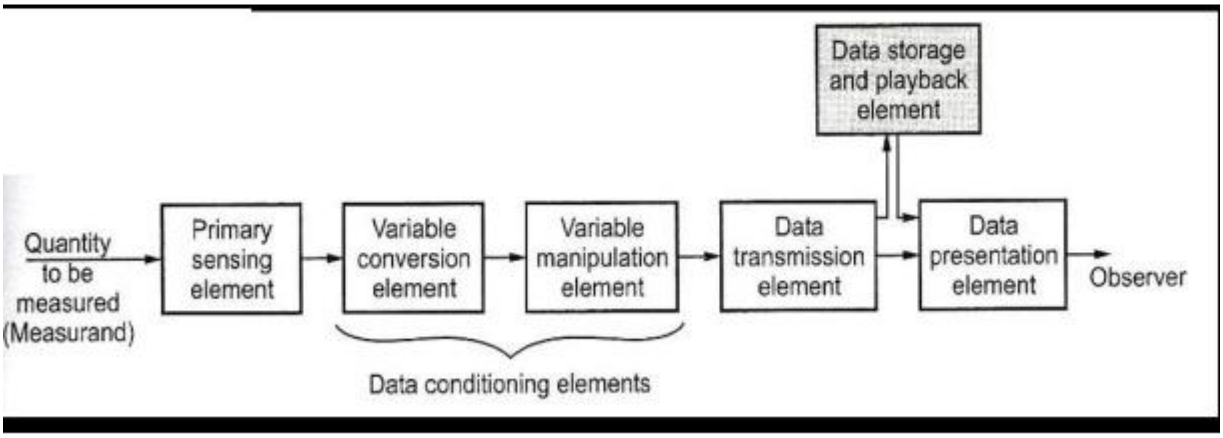
3. Electrical and Electronic measurements - Shawney, Khanna Publications.

4. Introduction to Instrumentation and measurements - Robert Northrop

UNIT I

Block Schematic of Measuring Systems

Block diagram of an instrument:



- Primary sensing element
- Variable conversion element
- Data presentation element

Primary sensing element

The quantity under measurement makes its first contact with primary sensing element of a measurement system here, the primary sensing element transducer. This transducer converts measured into an analogous electrical signal.

Variable conversion element

The output of the primary sensing element is the electrical signal. It may be a voltage a frequency or some other electrical parameter. But this output is not suitable for this system.

For the instrument to perform the desired function, it may be necessary to convert this output to some other suitable form while retaining the original signal. Consider an example, suppose output is an analog signal form and the next of system accepts input signal only in digital form .

Consider a small example, an electric amplifier circuit accepts a small voltage signal as input and produces an output signal which is also voltage but of greater amplifier. Thus voltage amplifier acts as a variable manipulation element.

Data presentation element

The information about the quantity under measurement has to be conveyed to the personal handling the instrument or system for control or analysis purposes. The information conveyed must be in the form of intelligible to the personnel. The above function is done by data presentation element.

The output or data of the system can be monitored by using visual display devices may be analog or digital device like ammeter, digital meter etc. In case the data to be record, we can use analog or digital recording equipment. In industries, force control and analysis purpose we can use computers.

The final stage in a measurement system is known as terminating stage . When a control device is used for the final measurement stage it is necessary to apply some feedback to the input signalto accomplish the control 0bjective.

The term signal conditioning includes many other functions in addition to variable conversion and variable manipulation. In fact the element that follows the primary sensing element in any instrument or instrumentation system should be called signal conditioning element.

When the element of an instrument is physically separated, it becomes necessary to transmit data from one to another. This element is called transmitting element. The signal conditioning and transmitting stage is generally known as intermediate stage.

Measurement system:

Measurement system any of the systems used in the process of associating numbers with physical quantities and phenomena. Although the concept of weights and measures today includes such factors as temperature, luminosity, pressure, and electric current, it once consisted of only four basic measurements: mass (weight), distance or length, area, and volume (liquid or grain measure). The last three are, of course, closely related. Basic to the whole idea of weights and measures are the concepts of uniformity, units, and standards.Uniformity, the essence of any system of weights and measures, requires accurate, reliable standards of mass and length.

Performance Characteristics

Static Characteristics of Instrument Systems:

Output/Input Relationship

Instrument systems are usually built up from a serial linkage of distinguishable building blocks. The actual physical assembly may not appear to be so but it can be broken down into a representative diagram of connected blocks. In the Humidity sensor it is activated by an input physical parameter and provides an output signal to the next block that processes the signal into a more appropriate state.

A key generic entity is, therefore, the relationship between the input and output of the block. As was pointed out earlier, all signals have a time characteristic, so we must consider the behavior of a block in terms of both the static and dynamic states.

The behavior of the static regime alone and the combined static and dynamic regime can be found through use of an appropriate mathematical model of each block. The mathematical description of system responses is easy to set up and use if the elements all act as linear systems and where addition of signals can be carried out in a linear additive manner. If nonlinearity exists in elements, then it becomes considerably more difficult — perhaps even quite impractical — to provide an easy to follow mathemat- ical explanation. Fortunately, general description of instrument systems responses can be usually be adequately covered using the linear treatment. The output/input ratio of the whole cascaded chain of blocks 1, 2, 3, etc. is given as:

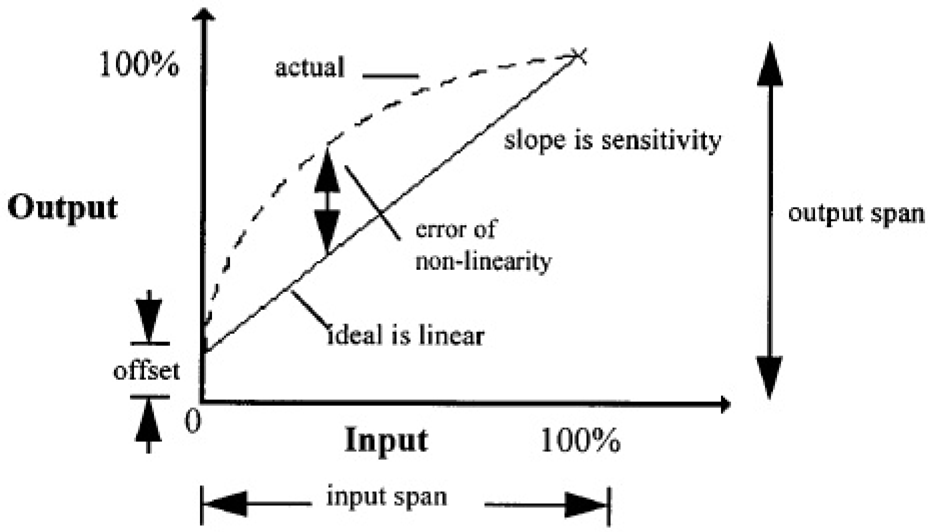
[output/input]total = [output/input]1× [output/input]2× [output/input]3 …

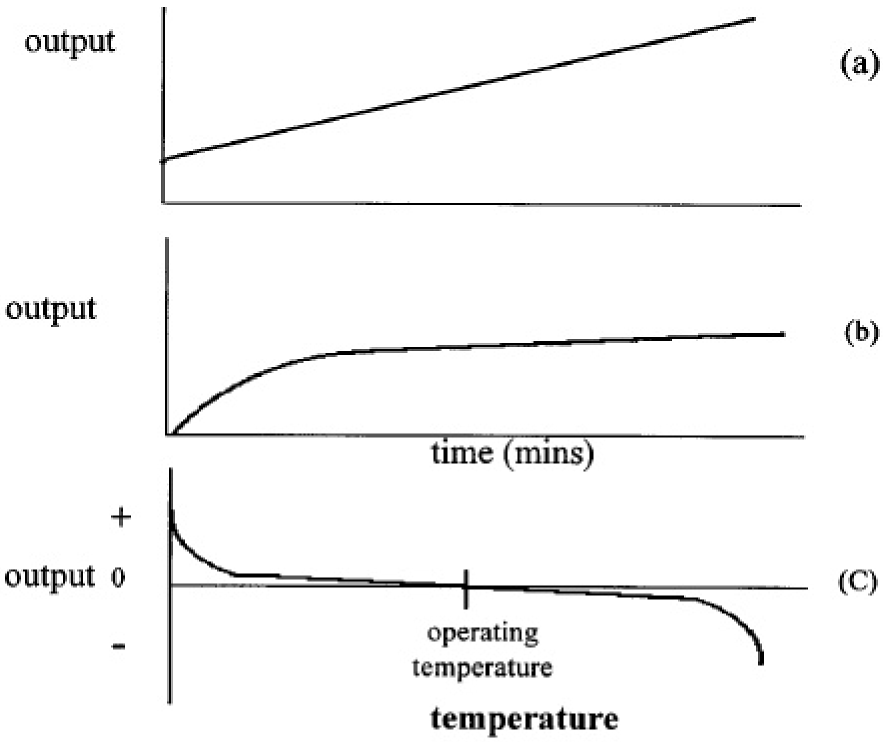
The output/input ratio of a block that includes both the static and dynamic characteristics is called the transfer function and is given the symbol G.

The equation forG can be written as two parts multiplied together. One expresses the static behavior of the block, that is, the value it has after all transient (time varying) effects have settled to their final state. The other part tells us how that value responds when the block is in its dynamic state. The static part is known as the transfer characteristic and is often all that is needed to be known for block description.

The static and dynamic response of the cascade of blocks is simply the multiplication of all individual blocks. As each block has its own part for the static and dynamic behavior, the cascade equations can be rearranged to separate the static from the dynamic parts and then by multiplying the static set and the dynamic set we get the overall response in the static and dynamic states. This is shown by the sequence of Equations.

Instruments are formed from a connection of blocks. Each block can be represented by a conceptual and mathematical model. This example is of one type of humidity sensor.





Drift :

It is now necessary to consider a major problem of instrument performance called instrument drift . This is caused by variations taking place in the parts of the instrumentation over time. Prime sources occur as chemical structural changes and changing mechanical stresses. Drift is a complex phenomenon for which the observed effects are that the sensitivity and offset values vary. It also can alter the accuracy of the instrument differently at the various amplitudes of the signal present.

Detailed description of drift is not at all easy but it is possible to work satisfactorily with simplified values that give the average of a set of observations, this usually being quoted in a conservative manner. The first graph (a) in Figure shows typical steady drift of a measuring

spring component of a weighing balance. Figure (b) shows how an electronic amplifier might settle down after being turned on.

Drift is also caused by variations in environmental parameters such as temperature, pressure, and humidity that operate on the components. These are known as influence parameters. An example is the change of the resistance of an electrical resistor, this resistor forming the critical part of an electronic amplifier that sets its gain as its operating temperature changes.

Unfortunately, the observed effects of influence parameter induced drift often are the same as for time varying drift. Appropriate testing of blocks such as electronic amplifiers does allow the two to be separated to some extent. For example, altering only the temperature of the amplifier over a short period will quickly show its temperature dependence.

Drift due to influence parameters is graphed in much the same way as for time drift. Figure shows the drift of an amplifier as temperature varies. Note that it depends significantly on the temperature

Drift in the performance of an instrument takes many forms:

(a) drift over time for a spring balance;

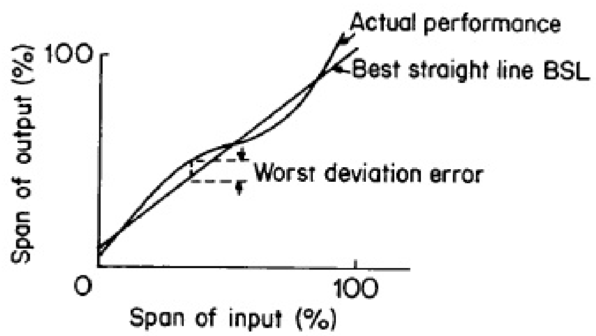
(b) how an electronic amplifier might settle over time to a final value after power is supplied;

(c) drift, due to temperature, of an electronic amplifier varies with the actual temperature of operation.

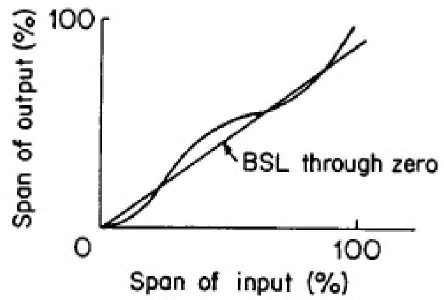
Dynamic Characteristics of Instrument Systems:

Dealing with Dynamic States:

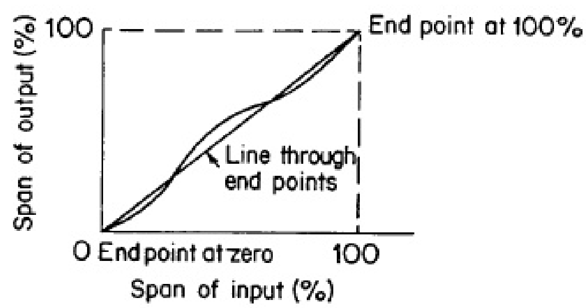
Measurement outcomes are rarely static over time. They will possess a dynamic component that must be understood for correct interpretation of the results. For example, a trace made on an ink pen chart recorder will be subject to the speed at which the pen can follow the input signal changes.Drift in the performance of an instrument takes many forms: (a) drift over time for a spring.



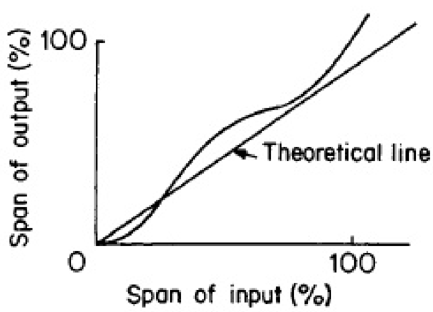
(a)



(b)



(c)



(d)

Error of nonlinearity can be expressed in four different ways: (a) best fit line (based on selected method used to decide this); (b) best fit line through zero; (c) line joining 0% and 100% points; and (d) theoretical line

To properly appreciate instrumentation design and its use, it is now necessary to develop insight into the most commonly encountered types of dynamic response and to develop the mathematical modeling basis that allows us to make concise statements about responses.

If the transfer relationship for a block follows linear laws of performance, then a generic mathematical method of dynamic description can be used. Unfortunately, simple mathematical methods have not been found that can describe all types of instrument responses in a simplistic and uniform manner. If the behavior is nonlinear, then description with mathematical models becomes very difficult and might be impracticable. The behavior of nonlinear systems can, however, be studied as segments of linear behavior joined end to end. Here, digital computers are effectively used to model systems of any kind provided the user is prepared to spend time setting up an adequate model.

Now the mathematics used to describe linear dynamic systems can be introduced. This gives valuable insight into the expected behavior of instrumentation, and it is usually found that the response can be approximated as linear.

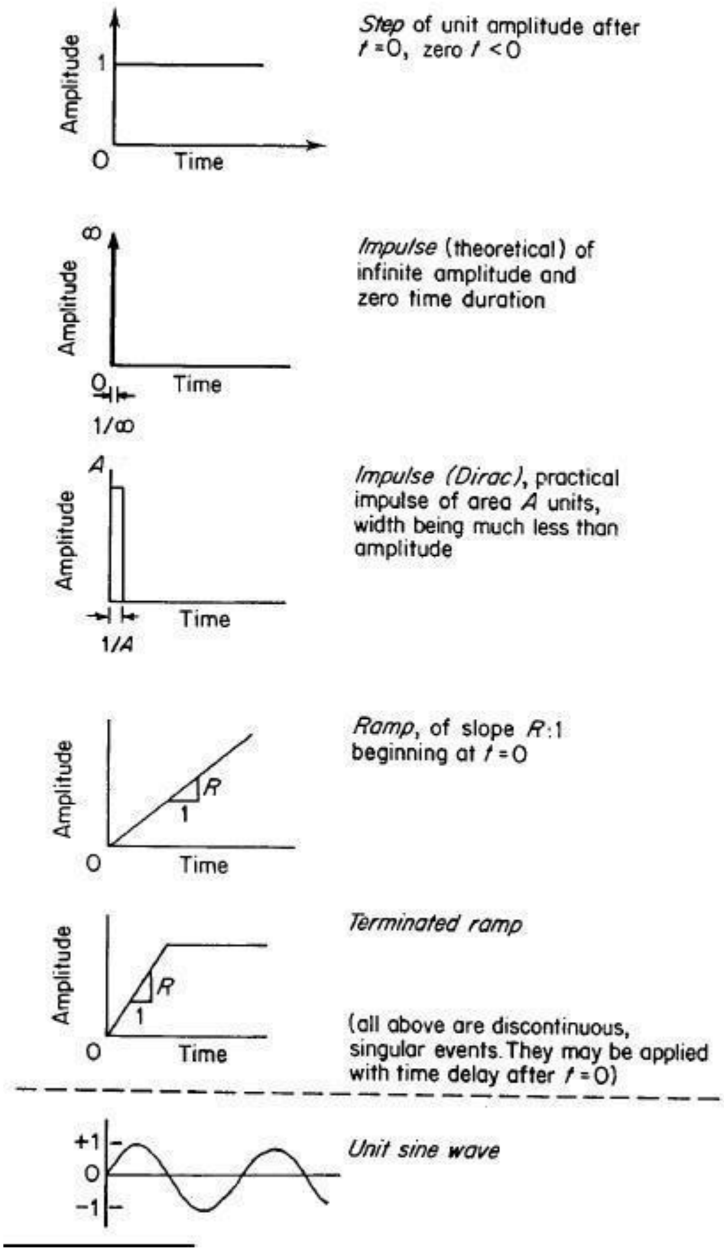
The modeled response at the output of a blockG_{result} is obtained by multiplying the mathematical expression for the input signalG_{input} by the transfer function of the block under investigation$G_{response}$, as shown in below equation.

$$G_{result} = G_{input} \times G_{response}$$

To proceed, one needs to understand commonly encountered input functions and the various types of block characteristics. We begin with the former set: the so-called forcing functions.

Forcing Functions

Let us first develop an understanding of the various types of input signal used to perform tests. The most commonly used signals are shown in Figure 3.12. These each possess different valuable test features. For example, the sine-wave is the basis of analysis of all complex wave-shapes because they can be formed as a combination of various sine-waves, each having individual responses that add to give all other wave- shapes. The step function has intuitively obvious uses because input transients of this kind are commonly encountered. The ramp test function is used to present a more realistic input for those systems where it is not possible to obtain instantaneous step input changes, such as attempting to move a large mass by a limited size of force. Forcing functions are also chosen because they can be easily described by a simple mathematical expression, thus making mathematical analysis relatively straightforward.



Unit of measurement:

A unit of measurement is a definite magnitude of a physical quantity, defined and adopted by convention and/or by law, that is used as a standard for measurement of the same physical quantity.[1] Any other value of the physical quantity can be expressed as a simple multiple of the unit of measurement.For example, length is a physical quantity. The metre is a unit of length that represents a definite predetermined length. When we say 10 metres (or 10 m), we actually mean 10 times the definite predetermined length called "metre".The definition, agreement, and practical use of units of measurement have played a

crucial role in human endeavour from early ages up to this day. Disparate systems of units used to be very common. Now there is a global standard, the International System of Units (SI), the modern form of the metric system.In trade, weights and measures is often a subject of governmental regulation, to ensure fairness and transparency. The Bureau international des poids et mesures (BIPM) is tasked with ensuring worldwide uniformity of measurements and their traceability to the International System of Units (SI). Metrology is the science for developing nationally and internationally accepted units of weights and measures.In physics and metrology, units are standards for measurement of physical quantities that need clear definitions to be useful. Reproducibility of experimental results is central to the scientific method. A standard system of units facilitates this. Scientific systems of units are a refinement of the concept of weights and measures developed long ago for commercial purposes.Science, medicine, and engineering often use larger and smaller units of measurement than those used in everyday life and indicate them more precisely. The judicious selection of the units of measurement can aid researchers in problem solving (see, for example, dimensional analysis).In the social sciences, there are no standard units of measurement and the theory and practice of measurement is studied in psychometrics and the theory of conjoint measurement.

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Types of Error:

Introduction

The knowledge we have of the physical world is obtained by doing experiments and making measurements. It is important to understand how to express such data and how to analyze and draw meaningful conclusions from it.In doing this it is crucial to understand that all measurements of physical quantities are subject to uncertainties. It is never possible to measure anything exactly. It is good, of course, to make the error as small as possible but it is always there. And in order to draw valid conclusions the error must be indicated and dealt with properly.Take the measurement of a person's height as an example. Assuming that her height has been determined to be 5' 8", how accurate is our result?Well, the height of a person depends on how straight she stands, whether she just got up (most people are slightly taller when getting up from a long rest in horizontal position), whether she has her shoes on, and how long her hair is and how it is made up. These inaccuracies could all be called errors of definition. A quantity such as height is not exactly defined without specifying many other circumstances.Even if you could precisely specify the “circumstances," your result would still have an error associated with it. The scale you are using is of limited accuracy; when you read the scale, you may have to estimate a fraction between the marks on the scale, etc.

If the result of a measurement is to have meaning it cannot consist of the measured value alone. An indication of how accurate the result is must be included also. Indeed, typically more effort is required to determine the error or uncertainty in a measurement than to perform the measurement itself. Thus, the result of any physical measurement has two essential components: (1) A numerical value (in a specified system of units) giving the best estimate possible of the quantity measured, and (2) the degree of uncertainty associated with this estimated value. For example, a measurement of the width of a table would yield a result such as 95.3 +/- 0.1 cm.

Significant Figures :

The significant figures of a (measured or calculated) quantity are the meaningful digits in it. There are conventions which you should learn and follow for how to express numbers so as to properly indicate their significant figures.

Any digit that is not zero is significant. Thus 549 has three significant figures and 1.892 has four significant figures.

Zeros between non zero digits are significant. Thus 4023 has four significant figures. Zeros to the left of the first non zero digit are not significant. Thus 0.000034 has only two significant figures. This is more easily seen if it is written as 3.4x10-5.

- For numbers with decimal points, zeros to the right of a non zero digit are significant. Thus 2.00 has three significant figures and 0.050 has two significant figures. For this reason it is important to keep the trailing zeros to indicate the actual number of significant figures.
- For numbers without decimal points, trailing zeros may or may not be significant. Thus, 400 indicates only one significant figure. To indicate that the trailing zeros
- are significant a decimal point must be added. For example, 400. has three significant figures, and 4×10^2 has one significant figure.

Exact numbers have an infinite number of significant digits. For example, if there are two oranges on a table, then the number of oranges is 2.000... . Defined numbers are also like this. For example, the number of centimeters per inch (2.54) has an infinite number of significant digits, as does the speed of light (299792458 m/s).

There are also specific rules for how to consistently express the uncertainty associated with a number. In general, the last significant figure in any result should be of the same order of magnitude (i.e.. in the same decimal position) as the uncertainty. Also, the uncertainty should be rounded to one or two significant figures. Always work out the uncertainty after finding the number of significant figures for the actual measurement.

For example,

9.82 +/- 0.02
10.0 +/- 1.5
4 +/- 1

The following numbers are all incorrect.

9.82 +/- 0.02385 is wrong but 9.82 +/- 0.02 is fine
10.0 +/- 2 is wrong but 10.0 +/- 2.0 is fine
4 +/- 0.5 is wrong but 4.0 +/- 0.5 is fine

In practice, when doing mathematical calculations, it is a good idea to keep one more digit than is significant to reduce rounding errors. But in the end, the answer must be expressed with only the proper number of significant figures. After addition or subtraction, the result is significant only to the place determined by the largest last significant place in the original numbers. For example,

89.332 + 1.1 = 90.432
should be rounded to get 90.4 (the tenths place is the last significant place in 1.1). After multiplication or division, the number of significant figures in the result is determined by the original number with the smallest number of significant figures. For example,

(2.80) (4.5039) = 12.61092

should be rounded off to 12.6 (three significant figures like 2.80).

Refer to any good introductory chemistry textbook for an explanation of the methodology for working out significant figures.

The Idea of Error :

The concept of error needs to be well understood. What is and what is not meant by "error"? A measurement may be made of a quantity which has an accepted value which can be looked up in a handbook (e.g.. the density of brass). The difference between the measurement and the accepted value is not what is meant by error. Such accepted values are not "right" answers. They are just measurements made by other people which have errors associated with them as well. Nor does error mean "blunder." Reading a scale backwards, misunderstanding what you are doing or elbowing your lab partner's measuring apparatus are blunders which can be caught and should simply be disregarded. Obviously, it cannot be determined exactly how far off a measurement is; if this could be done, it would be possible to just give a more accurate, corrected value. Error, then, has to do with uncertainty in measurements that nothing can be done about. If a measurement is repeated, the values obtained will differ and none of the results can be preferred over the others. Although it is not possible to do anything about such error, it can be characterized. For instance, the repeated measurements may cluster tightly together or they may spread widely. This pattern can be analyzed systematically.

Classification of Error :

Generally, errors can be divided into two broad and rough but useful classes: systematic and random.Systematic errors are errors which tend to shift all measurements in a systematic way so their mean value is displaced. This may be due to such things as incorrect calibration of equipment, consistently improper use of equipment or failure to properly account for some effect. In a sense, a systematic error is rather like a blunder and large systematic errors can and must be eliminated in a good experiment. But small systematic errors will always be present. For instance, no instrument can ever be calibrated perfectly.Other sources of systematic errors are external effects which can change the results of the experiment, but for which the corrections are not well known. In science, the reasons why several independent confirmations of experimental results are often required (especially using different techniques) is because different apparatus at different places may be affected by different systematic effects. Aside from making mistakes (such as thinking one is using the x10 scale, and actually using the x100 scale), the reason why experiments sometimes yield results which may be far outside the quoted errors is because of systematic effects which were not accounted for.

Random errors are errors which fluctuate from one measurement to the next. They yield results distributed about some mean value. They can occur for a variety of reasons.

- They may occur due to lack of sensitivity. For a sufficiently a small change an instrument may not be able to respond to it or to indicate it or the observer may not be able to discern it.
- They may occur due to noise. There may be extraneous disturbances which cannot be taken into account.
- They may be due to imprecise definition.
- They may also occur due to statistical processes such as the roll of dice.

Random errors displace measurements in an arbitrary direction whereas systematic errors displace measurements in a single direction. Some systematic error can be substantially eliminated (or properly taken into account). Random errors are unavoidable and must be lived with. Many times you will find results quoted with two errors. The first error quoted is usually the random error, and the second is called the systematic error. If only one error is quoted, then the errors from all sources are added together. (In quadrature as described in the section on propagation of errors.)A good example of "random error" is the statistical error associated with sampling or counting. For example, consider radioactive decay which occurs randomly at a some (average) rate. If a sample has, on average, 1000 radioactive decays per second then the expected number of decays in 5 seconds would be 5000. A particular measurement in a 5 second interval will, of course, vary from this average but it will generally yield a value within 5000 +/- . Behavior like this, where the error,

$$\Delta n = \sqrt{n_{expected}} \quad , (1)$$

$$n_{measured} = n_{expected}$$

is called a Poisson statistical process. Typically if one does not know $n_{expected}$ it is assumed that, in order to estimate this error.

Gaussian Error

,

Suppose an experiment were repeated many, say N, times to get,

$$x_1, x_2, \ldots x_k, \ldots x_N$$

N measurements of the same quantity, x. If the errors were random then the errors in these results would differ in sign and magnitude. So if the average or mean value of our measurements were calculated,

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_k + \ldots + x_N}{N} = \frac{\sum_{k=1}^{N} x_k}{N} \quad , (2)$$

some of the random variations could be expected to cancel out with others in the sum. This is the best that can be done to deal with random errors: repeat the measurement many times, varying as many "irrelevant" parameters as possible and use the average as the best estimate of the true value of x. (It should be pointed out that this estimate for a given N will differ from the limit as the true mean value; though, of course, for larger N it will be closer to the limit.) In the case of the previous example: measure the height at different times of day, using different scales, different helpers to read the scale, etc.Doing this should give a result with less error than any of the individual measurements. But it is obviously expensive, time consuming and tedious. So, eventually one must compromise and decide that the job is done. Nevertheless, repeating the experiment is the only way to gain confidence in and knowledge of its accuracy. In the process an estimate of the deviation of the measurements from the mean value can be obtained.

B. Root Sum Squares Formulae,

There are several different ways the distribution of the measured values of a repeated experiment such as discussed above can be specified.

- Maximum Error

The maximum and minimum values of the data set, x_{max} and x_{min}, could be specified. In these terms, the quantity,

$$\Delta x_{max} = \frac{x_{max} - x_{min}}{2}, (3)$$

is the maximum error. And virtually no measurements should ever fall outside $\bar{x} \pm \Delta x_{max}$.

- Probable Error

The probable error, Δx_{prob}, specifies the range $\bar{x} \pm \Delta x_{prob}$ which contains 50% of the measured values.

- Average Deviation

The average deviation is the average of the deviations from the mean,

$$\Delta x_{av} = \frac{\sum_k |x_k - \bar{x}|}{N}. (4)$$

For a Gaussian distribution of the data, about 58% will lie within $\bar{x} \pm \Delta x_{av}$.

- Standard Deviation

For the data to have a Gaussian distribution means that the probability of obtaining the result x is,

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-x_0)^2}{2\sigma^2}}, (5)$$

where x_0 is most probable value and σ, which is called the standard deviation, determines the width of the distribution. Because of the law of large numbers this assumption will tend to be valid for random errors. And so it is common practice to quote error in terms of the standard deviation of a Gaussian distribution fit to the observed data distribution. This is the way you should quote error in your reports.

It is just as wrong to indicate an error which is too large as one which is too small. In the measurement of the height of a person, we would reasonably expect the error to be +/-1/4" if a careful job was done, and maybe +/-3/4" if we did a hurried sample measurement. Certainly saying that a person's height is 5' 8.250"+/-0.002" is ridiculous (a single jump will compress your spine more than this) but saying that a person's height is 5' 8"+/- 6" implies that we have, at best, made a very rough estimate!

C. Standard Deviation

The mean is the most probable value of a Gaussian distribution. In terms of the mean, the standard deviation of any distribution is,

$$\sigma = \sqrt{\frac{\sum_k (x_k - \bar{x})^2}{N}} \quad . (6)$$

The quantity σ^2, the square of the standard deviation, is called the variance. The best estimate of the true standard deviation is,

$$\sigma_x = \sqrt{\frac{\sum_k (x_k - \bar{x})^2}{N-1}} \quad . (7)$$

The reason why we divide by N to get the best estimate of the mean and only by N-1 for the best estimate of the standard deviation needs to be explained. The true mean value of x is not being used to calculate the variance, but only the average of the measurements as the best estimate of it. Thus, $(x_k - \bar{x})^2$ as calculated is always a little bit smaller than $(x_k - \bar{x}_{true})^2$, the quantity really wanted. In the theory of probability (that is, using the assumption that the data has a Gaussian distribution), it can be shown that this underestimate is corrected by using N-1 instead of N.If one made one more measurement of x then (this is also a property of a Gaussian distribution) it would have some 68% probability of lying within $\bar{x} \pm \sigma_x$. Note that this means that about 30% of all experiments will disagree with the accepted value by more than one standard deviation.However, we are also interested in the error of the mean, which is smaller than sx if there were several measurements. An exact calculation yields,

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} = \sqrt{\frac{\sum_k (x_k - \bar{x})^2}{N(N-1)}} \quad , (8)$$

for the standard error of the mean. This means that, for example, if there were 20 measurements, the error on the mean itself would be = 4.47 times smaller then the error of each measurement. The number to report for this series of N measurements of x is $\bar{x} \pm \sigma_{\bar{x}}$ where $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$. The meaning of this is that if the N measurements of x were repeated there would be a 68% probability the new mean value of would lie within $\bar{x} \pm \sigma_{\bar{x}}$(that is between

$\bar{x}+\sigma_{\bar{x}}$ and $\bar{x}-\sigma_{\bar{x}}$). Note that this also means that there is a 32% probability that it will fall outside of this range. This means that out of 100 experiments of this type, on the average, 32 experiments will obtain a value which is outside the standard errors.

Examples :

Suppose the number of cosmic ray particles passing through some detecting device every hour is measured nine times and the results are those in the following table. Thus we have = 900/9 = 100 and σ_x^2 = 1500/8 = 188 or σ_x = 14. Then the probability that one more measurement of x will lie within 100 +/- 14 is 68%.The value to be reported for this series of measurements is 100+/-(14/3) or 100 +/- 5. If one were to make another series of nine measurements of x there would be a 68% probability the new mean would lie within the range 100 +/- 5.Random counting processes like this example obey a Poisson distribution for which $\sigma_x = \sqrt{\bar{x}}$. So one would expect the value of σ_x to be 10. This is somewhat less than the value of 14 obtained above; indicating either the process is not quite random or, what is more likely, more measurements are needed.

i		
1	80	400
2	95	25
3	100	0
4	110	100
5	90	100
6	115	225
7	85	225
8	120	400
9	105	25
S	900	1500

The same error analysis can be used for any set of repeated measurements whether they arise from random processes or not. For example in the Atwood's machine experiment to measure g you are asked to measure time five times for a given distance of fall s. The mean value of the time is,

$$\bar{t} = \frac{\Sigma t_i}{5} = \frac{t_1+t_2+t_3+t_4+t_5}{5} \quad , (9)$$

and the standard error of the mean is,

$$\sigma_{\bar{t}} = \sqrt{\frac{\sum (t_i - \bar{t})^2}{n(n-1)}} \text{, (10)}$$

where n = 5.

For the distance measurement you will have to estimate [[Delta]]s, the precision with which you can measure the drop distance (probably of the order of 2-3 mm).

Dynamic Characteristics:

Frequently, the result of an experiment will not be measured directly. Rather, it will be calculated from several measured physical quantities (each of which has a mean value and an error). What is the resulting error in the final result of such an experiment?

For instance, what is the error in Z = A + B where A and B are two measured quantities with errors and respectively?

A first thought might be that the error in Z would be just the sum of the errors in A and B. After all,

$$(A + \Delta A) + (B + \Delta B) = (A + B) + (\Delta A + \Delta B) \text{ (11)}$$

and

$$(A - \Delta A) + (B - \Delta B) = (A + B) - (\Delta A + \Delta B) \text{. (12)}$$

Repeatability

;

But this assumes that, when combined, the errors in A and B have the same sign and maximum magnitude; that is that they always combine in the worst possible way. This could only happen if the errors in the two variables were perfectly correlated, (i.e.. if the two variables were not really independent).If the variables are independent then sometimes the error in one variable will happen to cancel out some of the error in the other and so, on the average, the error in Z will be less than the sum of the errors in its parts. A reasonable way to try to take this into account is to treat the perturbations in Z produced by perturbations in its parts as if they were "perpendicular" and added according to the Pythagorean theorem,

$$\Delta Z = \sqrt{(\Delta A)^2 + (\Delta B)^2} \text{. (13)}$$

That is, if A = (100 +/- 3) and B = (6 +/- 4) then Z = (106 +/- 5) since $5 = \sqrt{3^2 + 4^2}$.

This idea can be used to derive a general rule. Suppose there are two measurements, A an

ΔA

will be perturbed by

$$\left(\frac{\partial F}{\partial A}\right)\Delta A$$

where (the partial derivative) [[partialdiff]]F/[[partialdiff]]A is the derivative of F with respect to A with B held constant. Similarly the perturbation in Z due to a perturbation in B is,

$$\left(\frac{\partial F}{\partial B}\right)\Delta B \, .$$

Combining these by the Pythagorean theorem yields

$$\Delta Z = \sqrt{\left(\frac{\partial F}{\partial A}\right)^2 (\Delta A)^2 + \left(\frac{\partial F}{\partial B}\right)^2 (\Delta B)^2} \, , (14)$$

In the example of Z = A + B considered above,

,

so this gives the same result as before. Similarly if Z = A - B then,

$$\frac{\partial F}{\partial A} = 1 \text{ and } \frac{\partial F}{\partial B} = -1 \, ,$$

which also gives the same result. Errors combine in the same way for both addition and subtraction. However, if Z = AB then,

$$\frac{\partial F}{\partial A} = B \text{ and } \frac{\partial F}{\partial B} = A \, ,$$

so

$$\Delta Z = \sqrt{B^2(\Delta A)^2 + A^2(\Delta B)^2} \, , (15)$$

Thus

$$\frac{\Delta Z}{z} = \frac{\Delta Z}{AB} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2} \, , (16)$$

or the fractional error in Z is the square root of the sum of the squares of the fractional errors in its parts. (You should be able to verify that the result is the same for division as it is for multiplication.) For example,

$$(100 \pm 0.3)(6 \pm 0.4) = 600 \pm 600\sqrt{\left(\frac{0.3}{100}\right)^2 + \left(\frac{0.4}{6}\right)^2} = 600 \pm 40$$

Reproducibility

It should be noted that since the above applies only when the two measured quantities are independent of each other it does not apply when, for example, one physical quantity is measured and what is required is its square. If Z = A2 then the perturbation in Z due to a perturbation in A is,

$$Z = \frac{\partial F}{\partial A}\Delta A = 2A\,\Delta A \quad . (17)$$

Thus, in this case,

$$(A \pm \Delta A)^2 = A^2 \pm 2A\,\Delta A = A^2\left(1 \pm 2\frac{\Delta A}{A}\right) \quad (18)$$

and not A2 (1 +/- ΔA/A) as would be obtained by misapplying the rule for independent variables. For example,

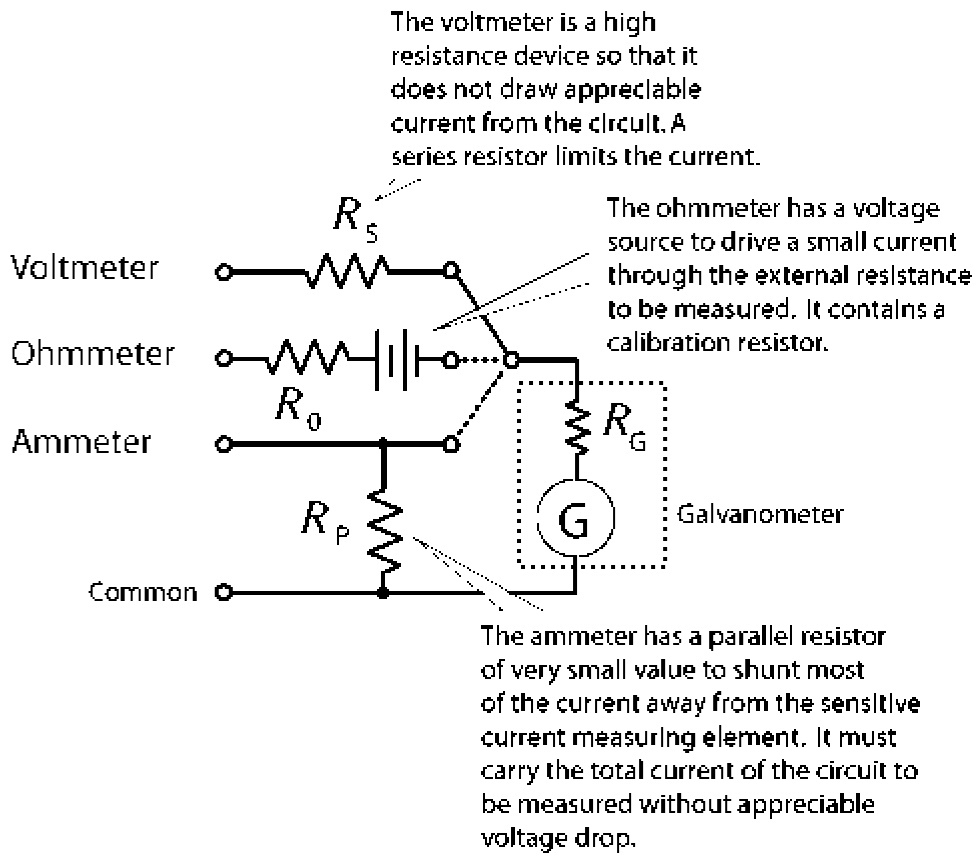
(10 +/- 1)2 = 100 +/- 20 and not 100 +/- 14.

If a variable Z depends on (one or) two variables (A and B) which have independent errors (ΔA and ΔB) then the rule for calculating the error in Z is tabulated in following table for a variety of simple relationships. These rules may be compounded for more complicated situations.

	Relation between Z and(A,B)	Relation between errors and (ΔA, ΔB)
1	Z = A + B	
2	Z = A - B	$(\Delta Z)^2 = (\Delta A)^2 + (\Delta B)^2$
3	Z = AB	$\left(\frac{\Delta Z}{Z}\right)^2 = \left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2$
4	Z = A/B	$\left(\frac{\Delta Z}{Z}\right)^2 = \left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2$
5	Z = An	$\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$
6	Z = ln A	$\Delta Z = \frac{\Delta A}{A}$
7	Z = eA	$\frac{\Delta Z}{Z} = \Delta A$

Fidelity, Lag

But this assumes that, when combined, the errors in A and B have the same sign and maximum magnitude; that is that they always combine in the worst possible way. This could only happen if the errors in the two variables were perfectly correlated, (i.e.. if the two variables were not really independent).If the variables are independent then sometimes the error in one variable will happen to cancel out some of the error in the other and so, on the average, the error in Z will be less than the sum of the errors in its parts. A reasonable way to try to take this into account is to treat the perturbations in Z produced by perturbations in its parts as if they were "perpendicular" and added according to the Pythagorean theorem,

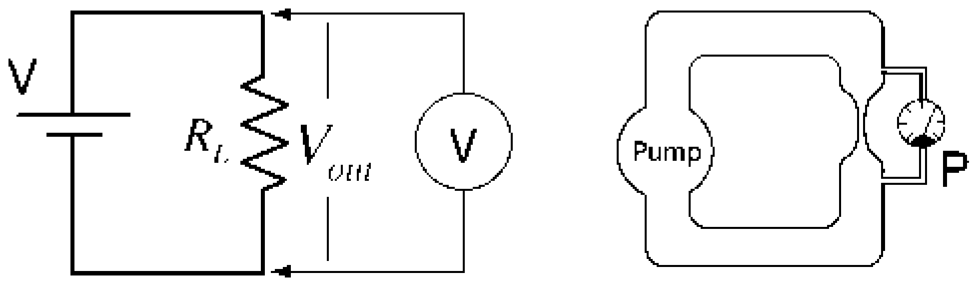


er. Since the
functions in a
single range

A voltmeter measures the change in voltage between two points in an electric circuit and therefore must be connected in parallel with the portion of the circuit on which the measurement is made. By contrast, an ammeter must be connected in series. In analogy with a water circuit, a voltmeter is like a meter designed to measure pressure difference. It is necessary for the voltmeter to have a very high resistance so that it does not have an appreciable affect on the current or voltage associated with the measured circuit. Modern solid-state meters have digital readouts, but the principles of operation can be better appreciated by examining the older moving coil meters based on galvanometer sensors.

DC Current Meters :

An ammeter is an instrument for measuring the electric current in amperes in a branch of an electric circuit. It must be placed in series with the measured branch, and must have very low resistance to avoid significant alteration of the current it is to measure. By contrast, an voltmeter must be connected in parallel. The analogy with an in-line flowmeter in a water circuit can help visualize why an ammeter must have a low resistance, and why connecting an ammeter in parallel can damage the meter. Modern solid-state meters have digital readouts, but the principles of operation can be better appreciated by examining the older moving coil metersbased on galvanometer sensors.



A voltmeter is connected in parallel to measure the voltage change across a circuit element.

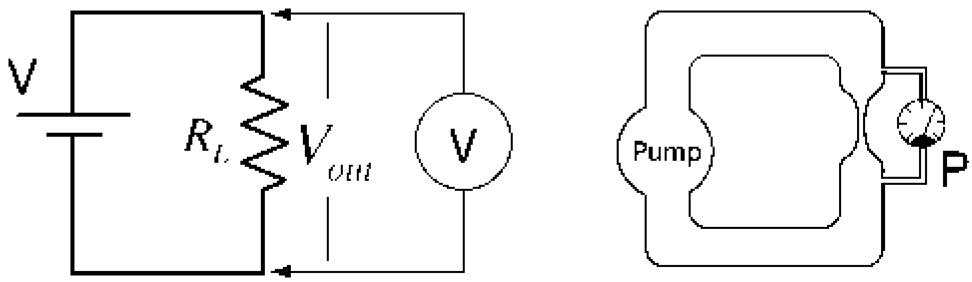
A pressure gauge is connected in parallel to measure the pressure drop across a region of resistance to flow.

A voltmeter is always connected in parallel with the part of the circuit for which you wish to measure voltage.

AC Voltmeters and Current Meters

The standard way to measure resistance in ohms is to supply a constant voltage to the resistance and measure the current through it. That current is of course inversely proportional to the resistance according to Ohm's law,so that you have a non-linear scale. The current registered by the current sensing element is proportional to 1/R, so that a large current implies a small resistance. Modern solid-state meters have digital readouts, but the principles of operation can be better appreciated by examining the older moving coil meters based on galvanometer sensors.

Ohmmeter :



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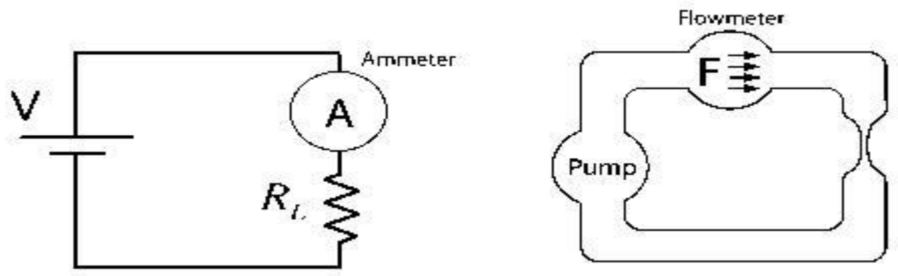
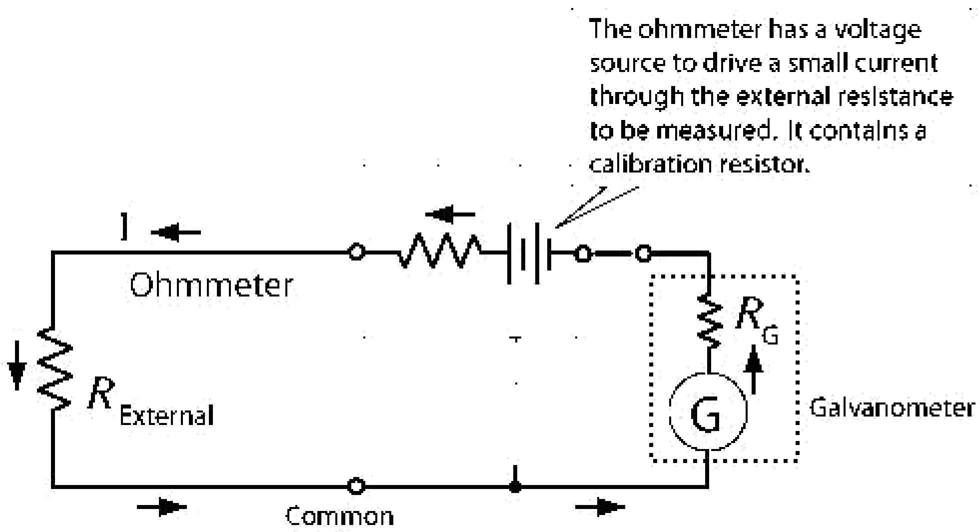


An ammeter is connected in series with a resistor to measure the current through the resistor.

A meter for volume flowrate must be in series to measure the flow, but must not appreciably affect the flow.

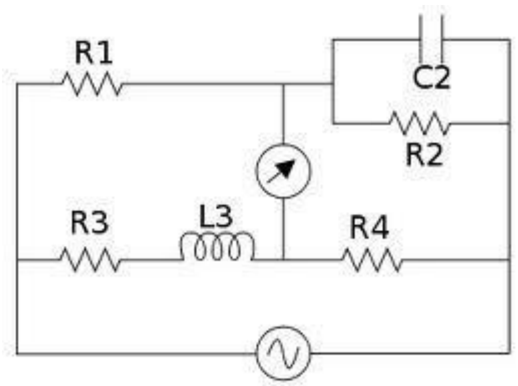


Multimeter, Meter Protection

RMS, or Root Mean Square, is the measurement used for any time varying signal's effective value: It is not an "Average" voltage and its mathematical relationship to peak voltage varies depending on the type of waveform. By definition, RMS Value, also called the effective or heating value of AC, is equivalent to a DC voltage that would provide the same amount of heat generation in a resistor as the AC voltage would if applied to that same resistor. Since an AC signal's voltage rises and falls with time, it takes more AC voltage to produce a given RMS voltage. In other words the grid must produce about 169 volts peak AC which turns out to be 120 volts RMS (.707 x 169). The heating value of the voltage available is equivalent to a 120 volt DC source (this is for example only and does not mean DC and AC are interchangeable). The typical multi-meter is not a True RMS reading meter. As a result it will only produce misleading voltage readings when trying to measure anything other than a DC signal or sine wave. Several types of multi-meters exist, and the owner's manual or the manufacturer should tell you which type you have. Each handles AC signals differently, here are the three basic types. A rectifier type multi-meter indicates RMS values for sine waves only. It does this by measuring average voltage and multiplying by 1.11 to find RMS. Trying to use this type of meter with any waveform other than a sine wave will result in erroneous RMS readings. Average reading digital volt meters are just that, they measure average voltage for an AC signal. Using the equations in the next column for a sine wave, average voltage (V_{avg}) can be converted to Volts RMS (V_{rms}), and doing this allows the meter to display an RMS reading for a sinewave.A True RMS meter uses a complex RMS converter to read RMS for any type of AC waveform.

, **Extension Of Range, True Rms Responding Voltmeters**,

To avoid the difficulties associated with determining the precise value of a variable capacitance, sometimes a fixed-value capacitor will be installed and more than one resistor will be made variable. The additional complexity of using a Maxwell bridge over simpler bridge types is warranted in circumstances where either the mutual inductance between the load and the known bridge entities, or stray electromagnetic interference, distorts the measurement results. The capacitive reactance in the bridge will exactly oppose the inductive reactance of the load when the bridge is balanced, allowing the load's resistance and reactance to be reliably determine



Specification Of Instruments.

Rx is the unknown resistance to be measured; R1, R2 and R3 are resistors of known resistance and the resistance of R2 is adjustable. If the ratio of the two resistances in the known leg (R2 / R1) is equal to the ratio of the two in the unknown leg (Rx / R3), then the voltage between the two midpoints (B and D) will be zero and no current will flow through the galvanometer Vg. R2 is varied until this condition is reached. The direction of the current indicates whether R2 is too high or too low.

Detecting zero current can be done to extremely high accuracy (see galvanometer). Therefore, if R1, R2 and R3 are known to high precision, then Rx can be measured to hi

precision. Very small changes in Rx disrupt the balance and are readily detected.At the point of balance, the ratio of R2 / R1 = Rx / R3

Therefore,

Alternatively, if R1, R2, and R3 are known, but R2 is not adjustable, the voltage difference across or current flow through the meter can be used to calculate the value of Rx, using Kirchhoff's circuit laws (also known as Kirchhoff's rules). This setup is frequently used in strain gauge and resistance thermometer measurements, as it is usually faster to read a voltage level off a meter than to adjust a resistance to zero the voltage.

$$I_1 - I_2 - I_g = 0$$

Then, Kirchhoff's second rule is used for finding the voltage in the loops ABD and BCD:

$$(I_3 \cdot R_3) - (I_g \cdot R_g) - (I_1 \cdot R_1) = 0$$
$$I_3 \cdot R_3 = I_1 \cdot R_1 R_2) + (I_g \cdot R_g) = 0$$

The bridge is balanced and Ig = 0, so the second set of equations can be rewritten as:

$$I_x \cdot R_x = I_2 \cdot R_2$$