UNIT-IV

WAVEGUIDES COMPONENTS-II

Ferrites

Ferrites are non-metallic insulators but with magnetic properties similar those of ferrous metals. They are a subgroup of ferromagnetic materials and ferrites that are widely used in microwave devices are manganese ferrite $MnFe_2O_3$ and Zinc ferrite $ZnFe_2O_3$. In addition to the above mentioned ferrites, another compound called Yttrium-Iron Garnet $Y_3Fe_2(FeO_4)_3$ or YIG in short which is actually a ferromagnetic material is also being used in the design of non reciprocal microwave devices.

The magnetic anisotropy is an important property of ferromagnetic materials and it is exhibited only upon the application of a bias, dc magnetic field. This field aligns the magnetic dipoles in the ferrite to produce a net non-zero magnetic dipole moment and causes these dipoles to precess at a frequency which depends upon the strength of the bias field. A microwave signal, circularly polarized, rotating in the direction same as this precession interacts strongly, while an oppositely rotating signal interacts lesser with the dipole moments. Since for a given direction of rotation, the sense of polarization changes with the direction of propagation, a microwave signal propagates through ferrite differently in different directions. This effect is utilized in the fabrication of directional devices such as isolators, circulators and gyrators.

Another useful characteristic is that the interaction with the applied microwave signal can be controlled by adjusting the strength of the bias magnetic field. This property is used in the design of phase shifters, switches, tunable resonators and filters.

$$\mathbf{B} = \mu\mathbf{H}$$

As the ferrites are anisotroic, their permeability is a tensor given by

$$\boldsymbol{\mu} = \begin{bmatrix} \mu & jK & 0 \\ -jK & \mu & 0 \\ 0 & 0 & \mu o \end{bmatrix}$$

Note that the tensor is an asymmetric one. A material having a permeability tensor of this form is called ‘gyrotropic’

The two properties of the ferrites which are important and relevant to microwave engineer are Faraday rotation and gyro-magnetic resonance.

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Faraday Rotation

A linearly polarized wave when propagates through the ferrite in the direction of bias, the polarization undergoes rotation proportional to the length of the ferrite. This phenomenon is called Faraday rotation. Faraday rotation is a non-reciprocal effect.

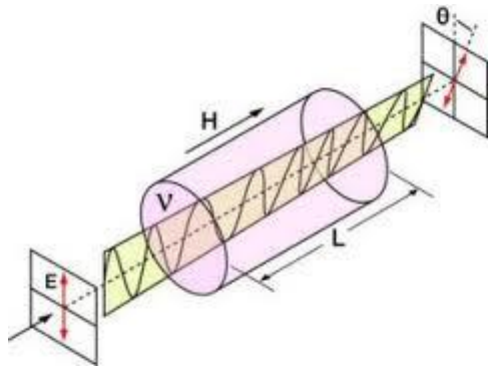
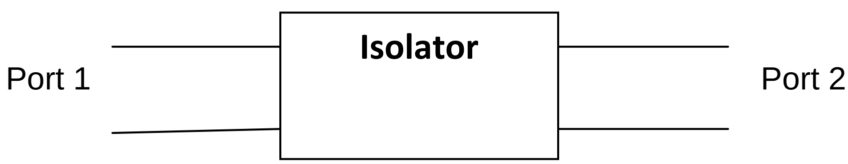


Fig 4.1: Faraday rotation

Isolator

Isolator is a two port non-reciprocal lossy device having unidirectional transmission characteristics.



The important aspects of this passive device are

1. When the wave propages from port 1 to port 2 there is no attenuation.
2. When the wave propages from port 1 to port 1 the attenuation is infinity.

The scattering matrix of isolator is

$$[S] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

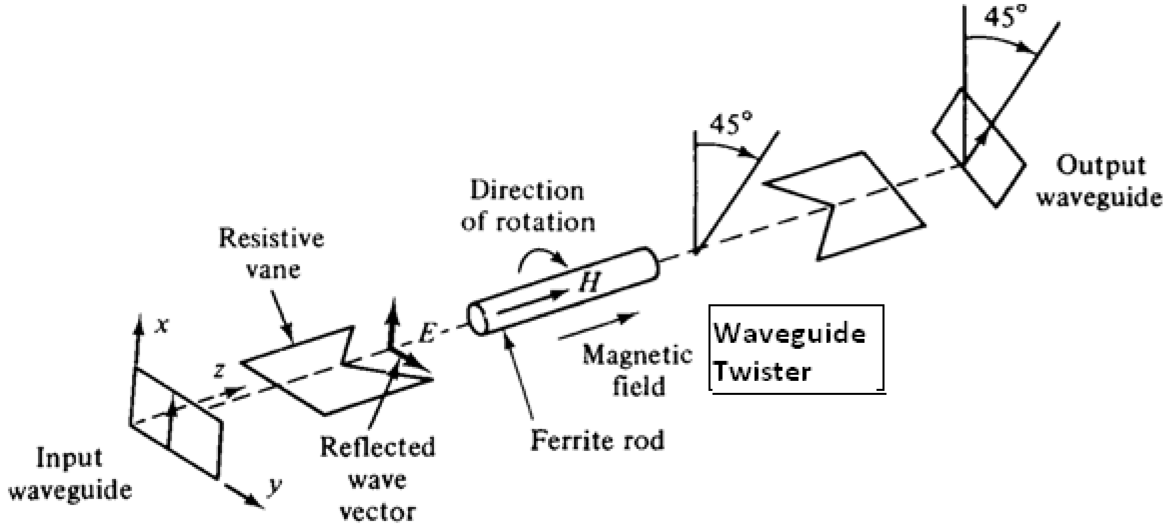


Fig 4.2: Isolator

The input card is in y-z plane¸ the dominant mode wave wherin the E-field vector is vertical travelling from left to right passes through resisitive vane with out attenduation and enters the ferrite rod where it undergoes faraday rotation of 45^0 clockwise. The wave again undergoes a rotation 45^0 in the anti-clockwise direction due to twisted waveguide and E-field vector at the output is vertical. The horizontal resistive vane has no effect on the E-field as the same is vertical to its plane. Therefore the wave travelling from left to right pass though without any attenuation.

Where as the dominant mode wave entering from right and travelling to left undergoes a rotation 45^0 in the anti-clockwise direction due to the twisted waveguide. As it passes though the ferrite rod it again undergoes a rotation 45^0 in the anti-clockwise direction and E-field vector becomes horizontal. The resistive vane at the output which is in the horiaontal plane obosbes the enegy as E-filed vecor is parallel to it. Therefore there is no output.

Gyrator

A gyrator is defined as a two-port device that has a relative difference in phase shift of 180^o for transmission from port 1 to port 2 as compared with phase shift for transmission from port 2 to port 1. A gyrator may be obtained by employing the nonreciprocal property of Faraday rotation. Figure given below illustrates a typical microwave gyrator. It consists of a rectangular guide with a 90^o twist connected to a circular guide. This in turn is connected to another rectangular guide at the other end. The two rectangular guides have the same orientation at the input ports. The circular guide contains a thin cylindrical rod of ferrite with the ends tapered to reduce reflections. A static axial magnetic field is applied so as to produce 90^o Faraday rotation to the TE_{11} dominant mode in the circular guide.

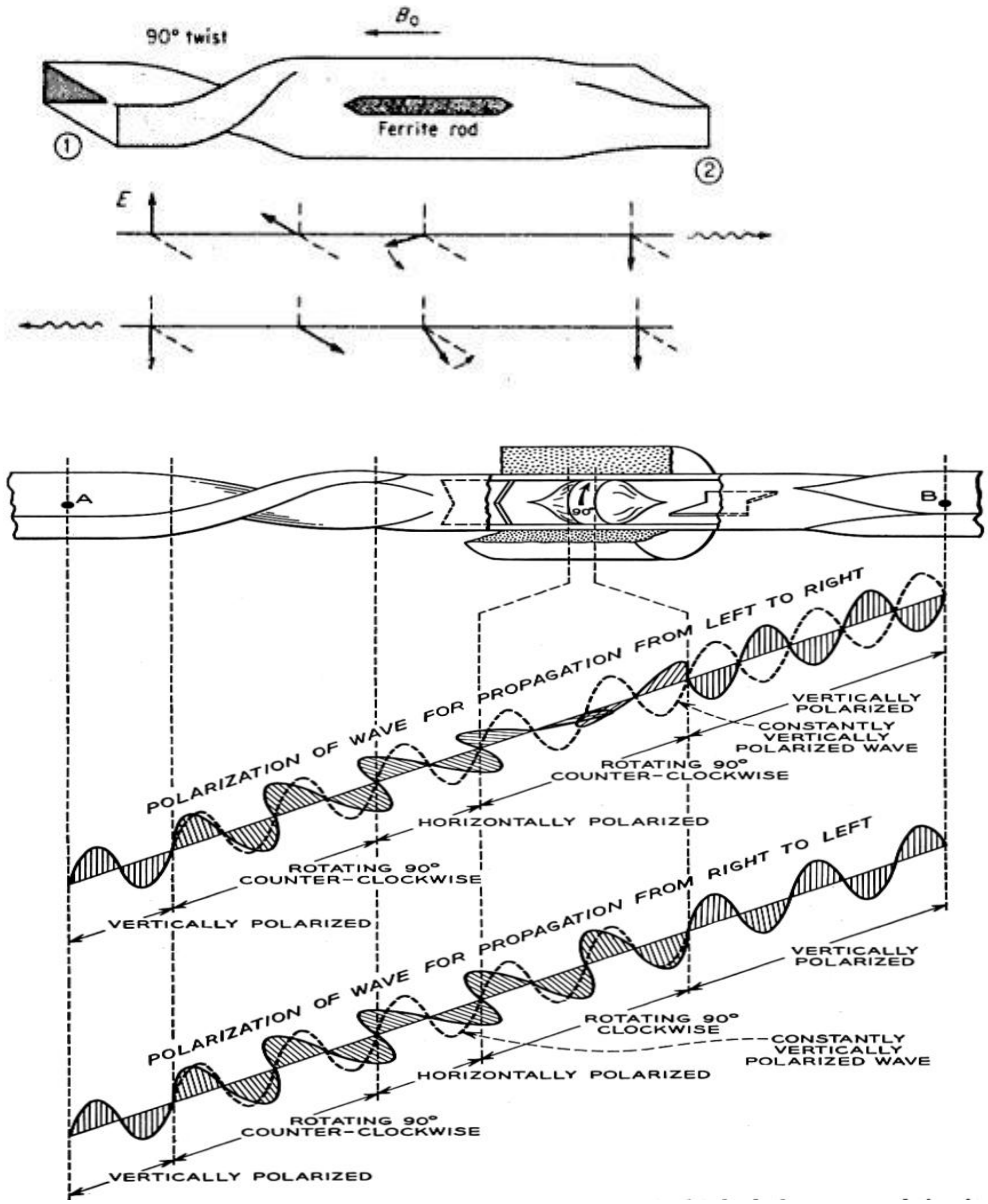


Fig 4.3: Gyrator

Consider as wave propagating from left to right. In passing through the twist the plane of polarization is rotated by 90^o in a counter clockwise direction. If the ferrite produced an additional 90^o of rotation, the total angle of rotation will be 180^o, as indicated in the figure above.

For a wave propagating from right to left, the Faraday rotation is still 90^o in the same sense. However, in passing through the twist, the next 90^o of rotation is in a direction to cancel the Faraday rotation. Thus for transmission from port 2 to port1, there is no phase shift.

Circulator

A microwave circulator is a multiport waveguide junction in which the wave can flow only from nth port to $(n+1)^{th}$ port in one direction. Please refer the figure given below. Although there is no restriction on the number of ports, four port microwave circulator is the most common.

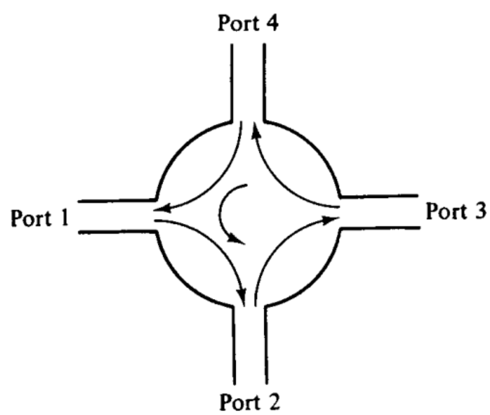


Fig 4.4: Four port Microwave Circulator

Many types of microwave circulators are in use today. However, their principles of operation remain the same. Figure given below shows a four port circulator constructed of two magic tees and a phase shifter. The phase shifter produces a phase shift of 180^o.

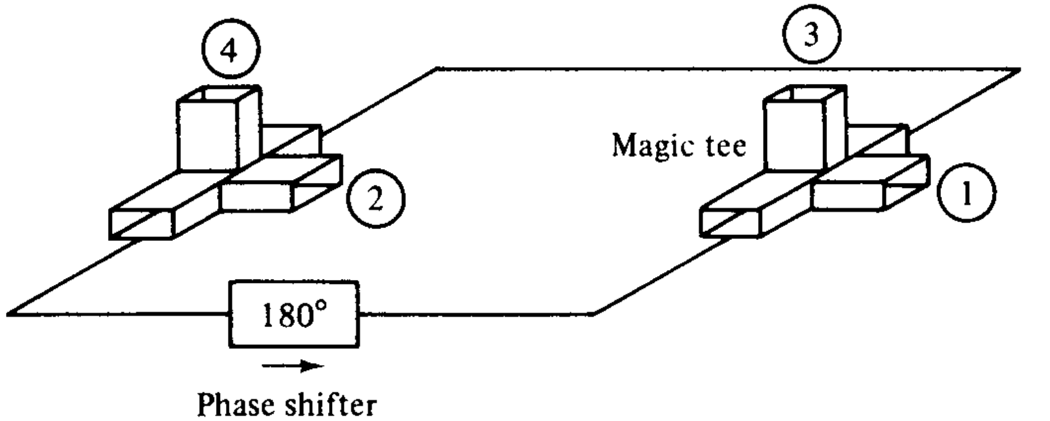


Fig 4.5: Circulator using 2 magic tees and one gyrator

Input from port 1: Gets splitted in two H-arms and enters the second magic Tee from right side path and left side path in phase, both gets cancelled in port 4 and gets added in port 2. Therefore output is available only from port 2.

Input from port 2: Gets splitted in two H-arms and enters the second magic Tee from right side path with 180^o phase shift and left side path with zero phase shift. Both the signals gets cancelled in port 2 and gets added in port 4. Therefore output is available only from port 4.

Input from port 4: Gets splitted in two E-arms and enters the second magic Tee from both the sides with in phase due to the gyrator. the signals gets cancelled in port 3 and gets added in port 1. Therefore output is available only from port 1.

Circulator using ferrite

Faraday rotation circulator consists of a piece of circular waveguide capable of carrying wave in the dominant mode TE_{11} with transitions to a standard rectangular guide which can carry TE_{10} at both the ends. The transition ports 1, 2 and two rectangular side ports 3 and 4 place with their broader wall along the length of the waveguide are twisted through 45^o. A thin ferrite rod is placed inside the circular waveguide supported by polyfoam and the waveguide is surrounded by a permanent magnet which produce dc magnetic field in the ferrite rod as shown below.

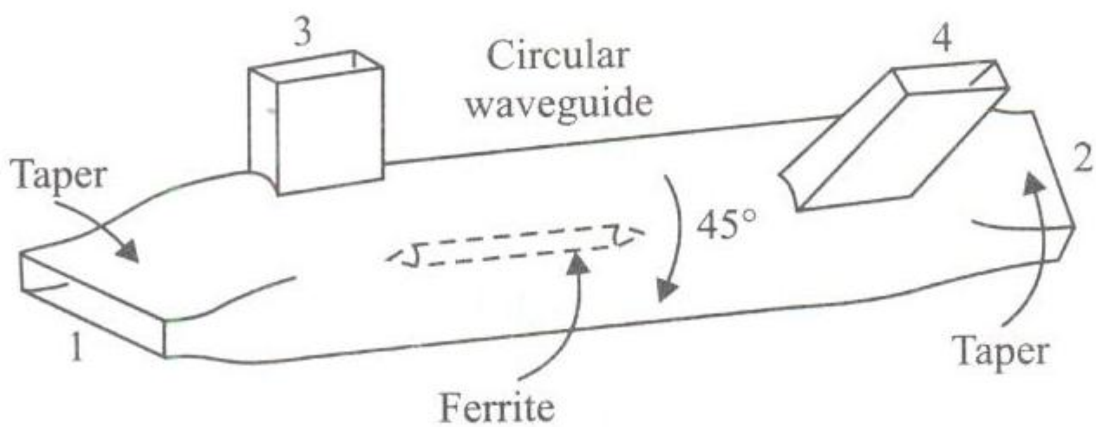


Fig 4.6: Circulator using Ferrite

Power fed from port 1: The wave travelling from port 1 passes port 3 unaffected as its electric field is not cut significantly gets rotated 45^o by the ferrite rod, continues past the port 4 unaffected reaching and emerging from the port 2 only.

Power fed from port 2: The wave travelling from port 2 passes port 4 unaffected as its electric field is not cut significantly gets rotated 45^o by the ferrite rod reaching and emerging from the port 3 only. In this case the wave cannot come out from port 1 because of shape and dimensions.

Power fed from port 3: It gets rotated 45^o and enters port 4 only.

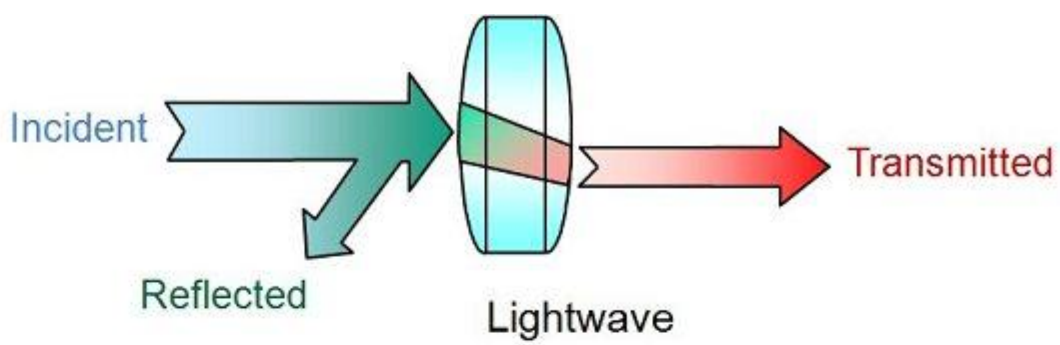
Power fed from port 3: It gets rotated 45^o and enters port 1 only.

S-parameters

The parameters are useful for electrical engineering, electronics engineering, and communication systems design, and especially for microwave engineering. The S-parameters are members of a family of similar parameters, other examples being:

- Y-parameters,
- Z-parameters,
- H-parameters,
- T-parameters or
- ABCD-parameters.

They differ from these, in the sense that *S-parameters* do not use open or short circuit conditions to characterize a linear electrical network; instead, matched loads are used. These terminations are much easier to use at high signal frequencies than open-circuit and short-circuit terminations. Moreover, the quantities are measured in terms of power.



Although applicable at any frequency, S-parameters are mostly used for networks operating at microwave frequencies where signal power and energy considerations are more easily quantified than currents and voltages. S-parameters change with the measurement frequency, so frequency must be specified for any S-parameter measurements stated, in addition to the characteristic impedance or system impedance.

S-parameters are readily represented in matrix form and obey the rules of matrix algebra.

Consider a two port network, as shown below and the network may be represented by S-parameters

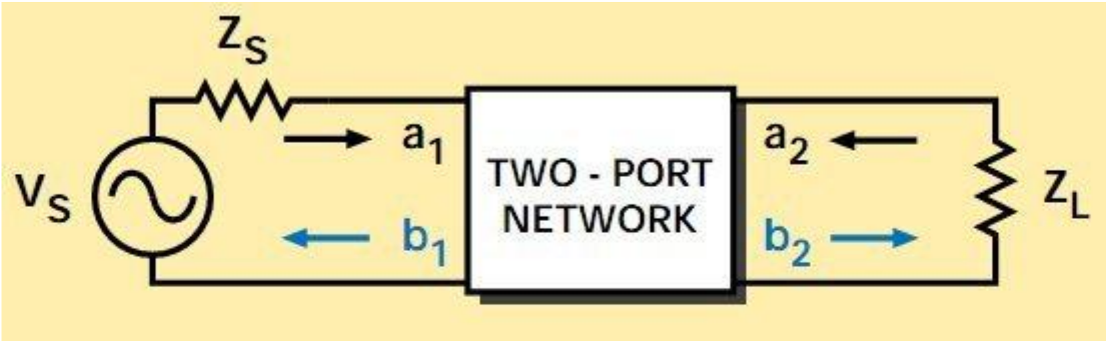


Figure 2 Two-port network showing incident waves (a1 , a2) and reflected waves (b1, b2) used in s-parameter definitions.

$$\begin{bmatrix} b1 \\ b2 \end{bmatrix} = \begin{bmatrix} s11 & s12 \\ s21 & s22 \end{bmatrix} \begin{bmatrix} a1 \\ a2 \end{bmatrix}$$

Where

$$|s_{11}|^2 = \frac{\text{Power reflected from the network input}}{\text{Power incident on the network input}}$$

$$|s_{22}|^2 = \frac{\text{Power reflected from the network output}}{\text{Power incident on the network output}}$$

$$|s_{21}|^2 = \frac{\text{Power delivered to a } Z_0 \text{ load}}{\text{Power available from } Z_0 \text{ source}}$$

$$= \text{Transducer power gain with } Z_0 \text{ load and source}$$

$$|s_{12}|^2 = \text{Reverse transducer power gain with } Z_0 \text{ load and}$$

$|a_1|^2$ = Power incident on the input of the network.
= Power available from a source impedance Z_0.

$|a_2|^2$ = Power incident on the output of the network.
= Power reflected from the load.

$|b_1|^2$ = Power reflected from the input port of the network.
= Power available from a Z_0 source minus the power delivered to the input of the network.

$|b_2|^2$ = Power reflected from the output port of the network.
= Power incident on the load.
= Power that would be delivered to a Z_0 load.

Another advantage of s-parameters springs from the simple relationship between the variables a_1, a_2 and b_1, b_2, and various power waves:

s-parameters are simply gains and reflection coefficients, both familiar quantities to engineers. At low frequencies most circuits behave in a predictable manner and can be described by a group of replaceable, lumped-equivalent black boxes. At microwave frequencies, as circuit element size approaches the wavelengths of the operating frequencies, such a simplified type of model becomes inaccurate. The physical arrangements of the circuit components can no longer be treated as black boxes. We have to use a distributed circuit element model and s-parameters.

The s-parameters s_{11}, s_{22}, s_{21}, and s_{12} are:

$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$ = Input reflection coefficient with the output port terminated by a matched load ($Z_L = Z_0$ sets $a_2 = 0$)

$$s_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$ = Output reflection coefficient with the input terminated by a matched load ($Z_S = Z_0$ sets $V_s = 0$)

$$s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$ = Forward transmission (insertion) gain with the output port terminated in a matched load.

$$s_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$ = Reverse transmission (insertion) gain with the input port terminated in a matched load.

$$b_1 = s_{11} a_1 + s_{12} a_2$$

$$b_2 = s_{21} a_1 + s_{22} a_2$$

The linear equations describing the two-port network are then:

The dependent variables b1 and b2 are normalized reflected voltages, as follows

$$b_1 = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}} = \frac{\text{voltage wave reflected from port 1}}{\sqrt{Z_0}} = \frac{V_{r1}}{\sqrt{Z_0}}$$

$$b_2 = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}} = \frac{\text{voltage wave reflected from port 2}}{\sqrt{Z_0}} = \frac{V_{r2}}{\sqrt{Z_0}}$$

$$a_1 = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}} = \frac{\text{voltage wave incident on port 1}}{\sqrt{Z_0}} = \frac{V_{i1}}{\sqrt{Z_0}}$$

$$a_2 = \frac{V_2 + I_2 Z_0}{2\sqrt{Z_0}} = \frac{\text{voltage wave incident on port 2}}{\sqrt{Z_0}} = \frac{V_{i2}}{\sqrt{Z_0}}$$

Properties of S-Parameters

(a) Zero Diagonal Elements for Perfect matched Network

For an ideal N-port matched network with matched termination at all the ports, $\mathbf{S_{ii} = 0}$ since there is no reflection from any port. Therefore, under perfect matched conditions, the diagonal elements of [S] are zero.

(b) Symmetry of [S] for a Reciprocal Network

A reciprocal device has the same transmission characteristics in either direction of a pair of ports and is characterized by a symmetric scattering matrix.

$$S_{ij} = S_{ji} \quad (i \neq j)$$

© Unitary Property for a Lossless Junction

For any lossless network, the sum of the products of each term of any one row or of any column of the S-matrix multiplied by its complex conjugate is unity.

$$[S]_t\,[S^*] = [U]$$

$$\text{Or} \quad [S^*] = [S]_t^{-1}$$

(d) Zero Property of S-Matrix

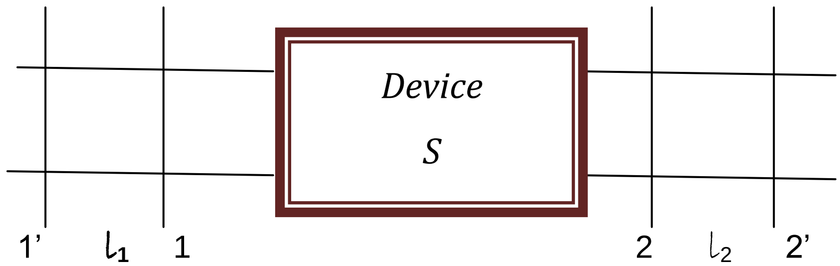
The sum of the products of each term of any column or row multiplied by the complex conjugate of the corresponding terms of any other column or row is zero.

$$S_{11}S_{12}^* + S_{21}S_{22}^* + \ldots\ldots\ldots\ldots + S_{N1}S_{N2}^* = 0$$

$$\sum_{j=1}^{N} Spj\ Sqj* = 0 \quad (p \neq q)$$

(e) Phase Shift Property

Complex S-parameters of network are defined with respect to the positions of the port or reference planes. For a two-port network with unprimed reference planes 1 and 2 as shown below, the S-parameters have definite complex values.



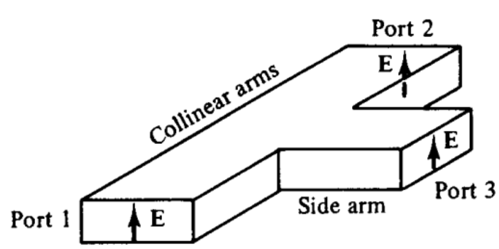
If the reference planes 1 and 2 are shifted outward to 1' and 2' by electrical phase $\Phi_1 = \beta_1 l_1$ and phase $\Phi_2 = \beta_2 l_2$ respectively, then the new wave variables are $a_1 e^{j\Phi 1}$, $b_1 e^{-j\Phi 1}$, $a_2 e^{j\Phi 2}$, $b_2 e^{-j\Phi 2}$ The new S-matrix S' is then given by

$$[S'] = \begin{bmatrix} \mathbf{e - j\Phi 1} & \mathbf{0} \\ \mathbf{0} & \mathbf{e - j\Phi 2} \end{bmatrix} [S] \begin{bmatrix} \mathbf{e - j\Phi 1} & \mathbf{0} \\ \mathbf{0} & \mathbf{e - j\Phi 2} \end{bmatrix}$$

S-matrix of H-plane Tee

The general S-matrix of a H Plane Tee junction is

$$[S] = \begin{bmatrix} S11 & S12 & S13 \\ S21 & S22 & S23 \\ S31 & S32 & S33 \end{bmatrix}$$



From considerations of symmetry and the phase relationship of the waves in each of the arms it can seen that a wave incident at Port 3 will result in waves at Ports 1 and 2, which are equal in magnitude and in phase, i.e.,

$$S_{31} = S_{13} = S_{23} = S_{32}$$

If two in-phase input waves are fed into ports 1 and 2 of the collinear arm, the output waves at port 3 will be in phase and additive.

Since port 3are electrically symmetrical with respect to port 1 and 2

$$S_{11} = S_{22}$$

All the diagonal elements of the S-matrix of an E-Plane Tee junction cannot be zero simultaneously since the tee junction cannot be matched to all three ports simultaneously. Considering Port 3 is matched the S-matrix of E-Plane Tee can be derived as follows.

$$S_{33} = 0$$

The S-Matrix then becomes

$$[S] = \begin{bmatrix} S11 & S12 & S13 \\ S12 & S11 & S13 \\ S13 & S13 & 0 \end{bmatrix}$$

From Unitary property

$$\begin{bmatrix} S11 & S12 & S13 \\ S12 & S22 & S13 \\ S13 & S13 & 0 \end{bmatrix} \begin{bmatrix} S11* & S12* & S13* \\ S12* & S22* & S13* \\ S13* & S13* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Considering the last row and last column

$$|S_{13}|^2 + |S_{13}|^2 = 1$$

$$|S_{13}| = 1/\sqrt{2}$$

By adjusting the reference plane at either Port1 or 3 the phase of S13 can be made zero. Therefore

$$S_{13} = 1/\sqrt{2}$$

The S-Matrix then becomes

$$[S] = \begin{bmatrix} S11 & S12 & 1/\sqrt{2} \\ S12 & S11 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

Now applying zero property to row 1 and row 3

$$|S_{11}|1/\sqrt{2} + |S_{12}|1/\sqrt{2} = 0$$

$$|S_{11}| = -|S_{12}|$$

The S-matrix now becomes

$$[S] = \begin{bmatrix} S11 & -S11 & 1/\sqrt{2} \\ -S11 & S11 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

From Unitary property

$$\begin{bmatrix} S11 & -S11 & 1/\sqrt{2} \\ -S11 & S11 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} S11* & -S11* & 1/\sqrt{2} \\ -S11* & S22* & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For R1 and C1

$$|S_{11}|^2 + |S_{11}|^2 + \left[\frac{1}{2}\right] = 1$$

$$|S_{11}| = 1/2$$

And $|S_{12}| = -1/2$

By adjusting the reference plane at either Port1 and / or 2 to make the phases of S_{11} and S_{12} zero

$$S_{12} = -S_{11} = -1/2$$

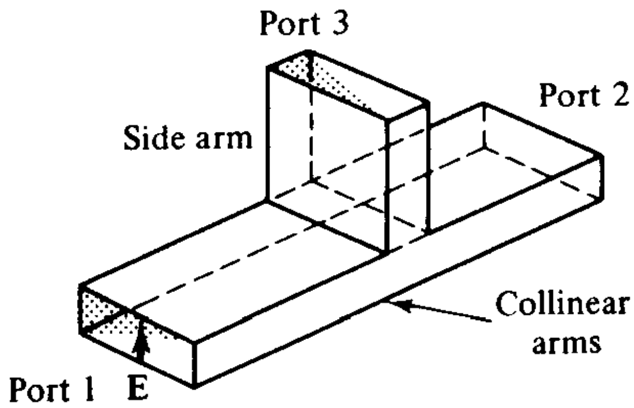
The S-matrix of H-plane Tee finally becomes

$$[S] = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

S-matrix of E-plane Tee

The general S-matrix of a Tee junction is

$$[S] = \begin{bmatrix} S11 & S12 & S13 \\ S21 & S22 & S23 \\ S31 & S32 & S33 \end{bmatrix}$$



From considerations of symmetry and the phase relationship of the waves in each of the arms it can seen that a wave incident at Port 3 will result in waves at Ports 1 and 2, which are equal in magnitude but opposite in phase, i.e.,

$$S_{31} = S_{13} = -S_{23} = -S_{32}$$

$$\text{and } S_{11} = S_{22}$$

If two in-phase input waves are fed into ports 1 and 2 of the collinear arm, the output waves at port 3 will be opposite in phase and subtractive.

Since port 1 and 2 are electrically symmetrical

$$S_{12} = S_{21}$$

All the diagonal elements of the S-matrix of an E-Plane Tee junction cannot be zero simultaneously since the tee junction cannot be matched to all three ports simultaneously. Considering Port 3 is matched the S-matrix of E-Plane Tee, i.e. $\mathbf{S_{33}=0}$, can be derived as follows.

The S-Matrix then becomes

$$[S] = \begin{bmatrix} S11 & S12 & S13 \\ S12 & S11 & -S13 \\ S13 & -S13 & 0 \end{bmatrix}$$

From Unitary property

$$\begin{bmatrix} S11 & S12 & S13 \\ S12 & S11 & -S13 \\ S13 & -S13 & 0 \end{bmatrix} \begin{bmatrix} S11* & S12* & S13* \\ S12* & S11* & -S13* \\ S13* & -S13* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Considering the last row and last column

$$|S_{13}|^2 + |S_{13}|^2 = 1$$

$$|S_{13}| = 1/\sqrt{2}$$

By adjusting the reference plane at either Port1 or 3 the phase of S13 can be made zero. Therefore

$$S_{13} = 1/\sqrt{2}$$

The S-matrix then becomes

$$[S] = \begin{bmatrix} S11 & S12 & 1/\sqrt{2} \\ S12 & S_{11} & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

Now applying zero property to row 1 and row 3

$$|S_{11}|1/\sqrt{2} - |S_{12}|1/\sqrt{2} = 0$$

This results in

$$|S_{11}| = |S_{12}|$$

The unitary property can now be stated as

$$\begin{bmatrix} S11 & S12 & 1/\sqrt{2} \\ S12 & S11 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} S11* & S12* & 1/\sqrt{2} \\ S12* & S22* & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying Unit property to R1 and C1

$$|S_{11}|^2 + |S_{11}|^2 + \left[\frac{1}{2}\right] = 1$$

$$|S_{11}|^2 + |S_{11}|^2 = \left[\frac{1}{2}\right]$$

$$|S_{11}| = |S_{12}| = 1/2$$

By adjusting the reference plane at Port1 and or 2 the phases of S_{11} and S_{12} can be made zero. Therefore

$$S_{11} = S_{12} = 1/2$$

Incorporating the above in S-matrix of E-Plane Tee becomes

$$[S] = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

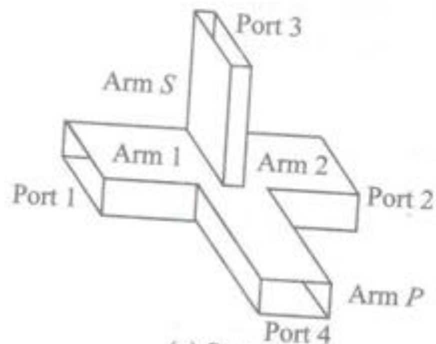
S-matrix of Magic Tee

The general S-matrix of a Tee junction is

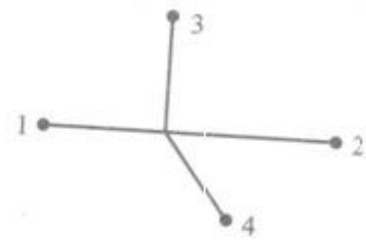
$$[S] = \begin{bmatrix} S11 & S12 & S13 & S14 \\ S21 & S22 & S23 & S24 \end{bmatrix}$$

$$\begin{matrix} S31 & S32 & S33 & S34 \\ S41 & S42 & S43 & S44 \end{matrix}$$

Consider the magic tee with the port designations as shown below



(a) Structure



In Magic Tee all four junctions can be perfectly matched, therefore

$$\mathbf{S_{11} = S_{22} = S_{33} = S_{44} = 0}$$

Port 1 and 2, Port 3 and 4 are isolated, therefore

$$\mathbf{S_{12} = S_{21} = S_{34} = S_{43} = 0}$$

Incorporating the above the S- matrix becomes

$$[S] = \begin{pmatrix} 0 & 0 & S13 & S14 \\ 0 & 0 & S23 & S24 \\ S31 & S32 & 0 & 0 \\ S41 & S42 & 0 & 0 \end{pmatrix}$$

As the ideal junction it is reciprocal. So the matrix is symmetrical. Port 3 is electrically anti-symmetrical with respect to the ports 1 and 2.

Therefore $\mathbf{S_{23} = -S_{13} = S_{32} = -S_{31}}$

Port 4 is electrically symmetrical with respect to the ports 1 and 2. Therefore

$$\mathbf{S_{24} = S_{14} = S_{41} = S_{42}}$$

Incorporating the above the S- matrix becomes

$$[S] = \begin{pmatrix} 0 & 0 & S13 & S14 \\ 0 & 0 & -S13 & S14 \\ S13 & -S13 & 0 & 0 \\ S14 & S14 & 0 & 0 \end{pmatrix}$$

The Unitary property becomes

$$\begin{pmatrix} 0 & 0 & S13 & -S13 \\ 0 & 0 & -S13 & S14 \\ S13 & -S13 & 0 & 0 \\ S14 & S14 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & S13^* & S14^* \\ 0 & 0 & -S13^* & S14^* \\ S13^* & -S13^* & 0 & 0 \\ S14^* & S14^* & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Applying Unit property to R3 and C3

$$|S_{13}|^2 + |S_{13}|^2 = 1$$

$$|S_{13}| = 1/\sqrt{2}$$

By adjusting the reference plane at either Port 1 or 3 the phase of S13 can be made zero. Therefore

$$S_{13} = 1/\sqrt{2}$$

The S-matrix now becomes

$$[S] = \begin{pmatrix} 0 & 0 & 1/\sqrt{2} & S14 \\ 0 & 0 & -1/\sqrt{2} & S14 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ S14 & S14 & 0 & 0 \end{pmatrix}$$

The Unitary property becomes

$$\begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & S14 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ S14 & S14 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & S14^* \\ 0 & 0 & -\frac{1}{\sqrt{2}} & S14^* \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ S14^* & S14^* & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Applying Unit property to R4 and C4

$$|S_{14}|^2 + |S_{14}|^2 = 1$$

$$|S_{14}| = 1/\sqrt{2}$$

By adjusting the reference plane at either Port 1 or 4 the phase of S14 can be made zero. Therefore

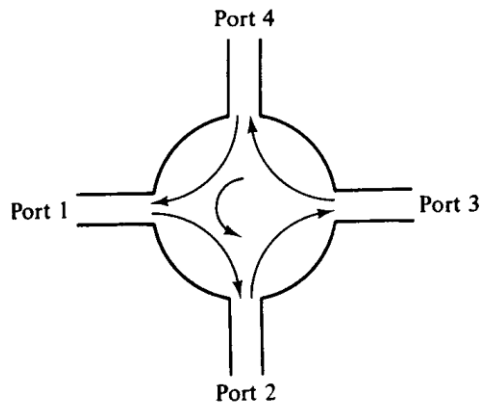
$$S_{14} = 1/\sqrt{2}$$

Thus the S-matrix of Magic Tee is finally given by

$$[S] = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

S-Matrix of Circulator

Circulators are n-port lossless and non-reciprocal device matched at all the ports in which power flow occur from ports 1 to 2, port 2 to 3,….., Port n-1 to n, Port n to 1.



Consider a four port circulator as shown above, which is a commonly use device.

The general S-matrix of a Circulator is

$$[S] = \begin{pmatrix} S11 & S12 & S13 & S14 \\ S21 & S22 & S23 & S24 \\ S31 & S32 & S33 & S34 \\ S41 & S42 & S43 & S44 \end{pmatrix}$$

Form the properties of circulator; all the ports are perfectly matched.

$$\mathbf{S_{11} = S_{22} = S_{33} = S_{44} = 0}$$

Considering the ports are reciprocal,

$$\mathbf{S_{21} = S_{32} = S_{43} = S_{14}}$$

Rest of the S-matrix elements are zero.(S_{12}=S_{13}=S_{23}= S_{24}=S_{31}=S_{34}= S_{41}=S_{42} =0)

The S-matrix becomes

$$[S] = \begin{pmatrix} 0 & 0 & 0 & S21 \\ S21 & 0 & 0 & 0 \\ 0 & S21 & 0 & 0 \\ 0 & 0 & S21 & 0 \end{pmatrix}$$

The unitary property is

The Unitary property becomes

$$\begin{pmatrix} \mathbf{0} & \boldsymbol{S21} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{S21} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{S21} \\ \boldsymbol{S21} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{S21*} \\ \boldsymbol{S21*} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{S21*} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{S21*} & \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

Applying Unit property to R4 and C4

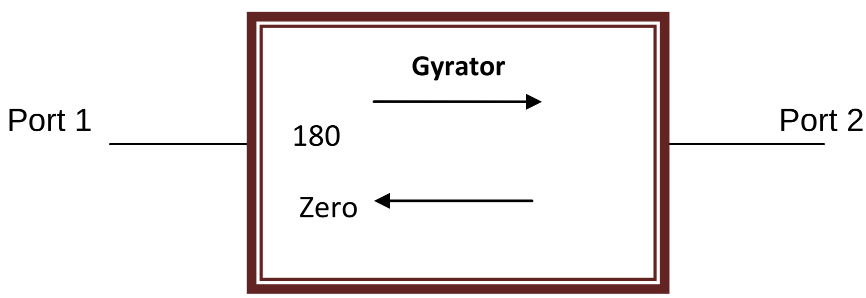
$$|S_{21}|^2 = 1$$

Therefore $\mathbf{S_{21} = 1}$

The final S-matrix of Circulator becomes

$$[S] = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{pmatrix}$$

S-Matrix of Gyrator



Gyrator are the two port devices which are non- reciprocal having 180^o differential phase shift.

If the two ports are perfectly matched $\mathbf{S_{11} = S_{22} = 0}$

From the property of Gyrator **S12= - S21**

The S-matrix becomes

$$[S] = \begin{bmatrix} 0 & S12 \\ -S12 & 0 \end{bmatrix}$$

From the unitary property

$$\begin{bmatrix} 0 & -S12 \\ S12 & 0 \end{bmatrix} \begin{bmatrix} 0 & S12* \\ -S12* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Multiply R2 with C2

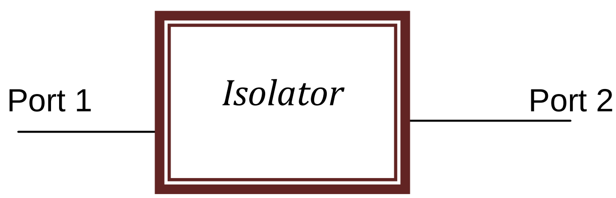
$\mathbf{|S_{12}|^2 = 1}$

Therefore $\mathbf{S_{12} = 1}$

The Gyrator S-matrix finally becomes

$$\mathbf{[S] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}}$$

S-Matrix of Isolator



Isolator are the two port devices which are non- reciprocal having no attenuation from port 1 to port 2 and infinite attenuation from port 2 to port 1.

If the two ports are perfectly matched $\mathbf{S_{11} = S_{22} = 0}$

From the property of isolator $\mathbf{S_{12} = 0}$

The S-matrix becomes

$$[S] = \begin{bmatrix} 0 & 0 \\ S21 & 0 \end{bmatrix}$$

From the unitary property

$$\begin{bmatrix} 0 & S21 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ S21* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Multiply R1 with C1

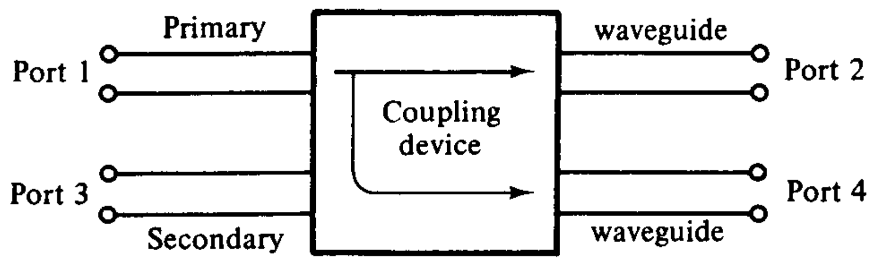
$\mathbf{|S_{21}|^2 = 1}$

Therefore $\mathbf{S_{21} = 1}$

The Isolator S-matrix finally becomes

$$[S] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

S-matrix of Directional Coupler



A directional coupler is a 4-port device therefore the matrix can be written as

$$[S] = \begin{bmatrix} S11 & S12 & S13 & S14 \\ S21 & S22 & S23 & S24 \\ S31 & S32 & S33 & S34 \\ S41 & S42 & S43 & S44 \end{bmatrix}$$

Form the properties of circulator; all the ports are perfectly matched. Thus the diagonal elements become zero.

$$S_{11} = S_{22} = S_{33} = S_{44} = 0$$

As there is no coupling between port1 and port 3 then between port 2 and port 4

$$S_{13} = S_{31} = S_{24} = S_{42} = 0$$

Since the ports are reciprocal

$$S_{12} = S_{21},\ S_{14} = S_{41},\ S_{23} = S_{32},\ S_{34} = S_{43}$$

Consequently the S-matrix becomes

$$[S] = \begin{bmatrix} 0 & S12 & 0 & S14 \\ S21 & 0 & S23 & 0 \\ 0 & S32 & 0 & S34 \\ S41 & 0 & S43 & 0 \end{bmatrix}$$

Form the properties of circulator; all the ports are perfectly matched. Thus the diagonal elements become zero.

As the coupler is ideal and its is reciprocal, the matrix must be symmetrical
From the unitary property

Consequently the S-matrix becomes

$$[S] = \begin{bmatrix} 0 & S12 & 0 & S14 \\ S12 & 0 & S23 & 0 \\ 0 & S23 & 0 & S34 \\ S14 & 0 & S34 & 0 \end{bmatrix}$$

The Unitary property becomes

$$\begin{bmatrix} 0 & S12 & 0 & S14 \\ S12 & 0 & S23 & 0 \\ 0 & S23 & 0 & S34 \\ S14 & 0 & S34 & 0 \end{bmatrix} \begin{bmatrix} 0 & S12* & 0 & S14* \\ S12* & 0 & S23* & 0 \\ 0 & S23* & 0 & S34* \\ S14* & 0 & S34* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and multiplying R2 with C4

$$S_{12} S_{14}^* + S_{23} S_{34}^* = 0 \qquad (4.1)$$

or $$|S_{12}||S_{14}| = |S_{23}||S_{34}| \qquad (4.2)$$

multiplying R1 with C3

$$S_{21} S_{23}^* + S_{14} S_{34}^* = 0 \qquad (4.3)$$

or $$|S_{21}||S_{23}| = |S_{14}||S_{34}| \qquad (4.4)$$

Multiplying R1 with C1

$$S_{12} S_{12}^* + S_{14} S_{14}^* = |S_{12}|^2 + |S_{14}|^2 = 1 \qquad (4.5)$$

As $S_{23} = S_{32}$, from 4.2

$$|S_{12}||S_{14}| = |S_{32}||S_{34}| = |S_{23}||S_{34}| \qquad (4.6)$$

and from 4.4

$$|S_{21}||S_{23}| = |S_{41}||S_{43}| \qquad (4.7)$$

or $$|S_{12}||S_{23}| = |S_{14}||S_{34}| \qquad (4.8)$$

Multiply eqns 4.6 and 4.8

$$|S_{12}|^2 |S_{14}||S_{23}| = |S_{14}||S_{23}||S_{34}|^2$$

Therefore $|S_{12}| = |S_{34}|$ (4.9)

Similarly from eqns 4.6 and 4.9

$$|S_{14}| = |S_{23}| \quad (4.10)$$

Let $S_{12} = S_{34} = p$, where **p** is positive and real

Then from eqn 4.3, we can write

$$p(S_{23}^* + S_{41}) = 0 \quad (4.11)$$

Let $S_{23} = S_{14} = jq$, where **q** is positive and real

From eqn 4.5 we can write

$$p^2 + q^2 = 1$$

Consequently the S-matrix becomes

$$[S] = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$$