UNIT-VI

HELIX TWT AND M-TYPE TUBES

Helix Travelling Wave Tubes

In the previous unit klystrons and reflex klystrons were analysed in some detail. When it comes to study of TWTs it is appropriate to compare the basic operating principles of both TWT and the klystron. In the case of TWT, the microwave circuit is non-resonant and the wave propagates with the same speed as the electrons in the beam. The initial effect on the beam is a small amount of velocity modulation caused by the weak electric fields associated with the traveling wave. Just as in the klystron, this velocity modulation later translates to current modulation, which then induces an RF current in the circuit, causing amplification. However, there are some major differences between the TWT and the klystrons.

1. The interaction of electron beam and RF field in the TWT is continuous over the entire length of the circuit, but the interaction in the klystron occurs only at the gaps of a few resonant cavities.
2. The wave in the TWT is a propagating wave; the wave in the klystron is not.
3. In the couple cavity TWT there is a coupling effect between the cavities, whereas each cavity in the klystron operates independently.

A **traveling-wave tube (TWT)** is a Microwave Amplifier with following characteristics:

1. Low Power Amplifier: up to 10 kW
2. Frequency Range: 3 G Hz – 50 G Hz
3. Wide Band width: about 800 MHz
4. Power gain: upto 60 dB
5. Efficiency: 20 – 40 %

Comparison between TWT and Klystron Amplifier

S.No.	Klystron Amplifier	TWT Amplifier
1	Linear beam ‘O’ Type	Linear beam ‘O’ Type
2	Uses input and output resonant cavities	Uses non-resonant wave circuit
3	Narrow band amplifier	Wide band amplifier BW 800 MHz
4	Interaction between electrons and the field is very short	Longer interaction
5	Non propagating wave	Propagating wave

The TWT operates on the principle of slow wave. It is a not resonant ‘O’-Type microwave device. Its operation is base on the interaction between the waves in the travelling wave structure and the electronic beam. The main elements of the TWT Amplifier are:

(1) Electron gun;
(2) RF input
(3) Magnets
(4) Attenuator
(5) Helix coil
(6) RF output
(7) Vacuum tube
(8) Collector

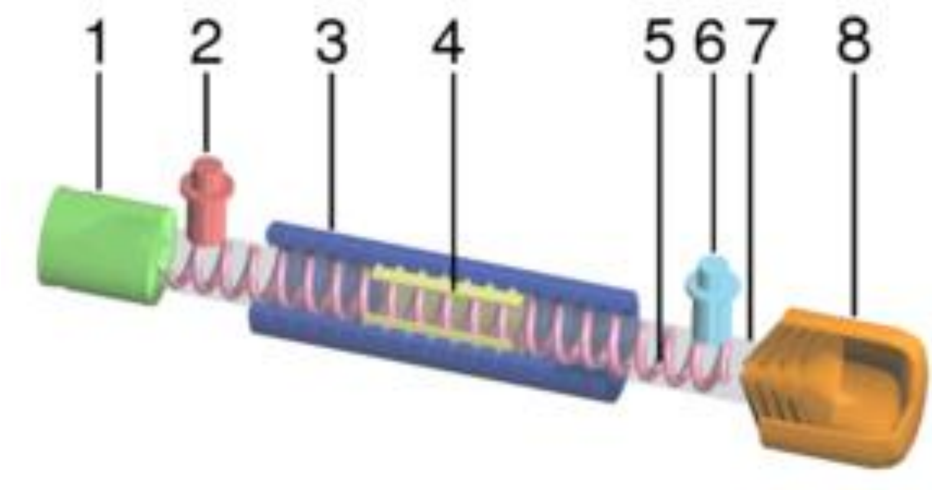
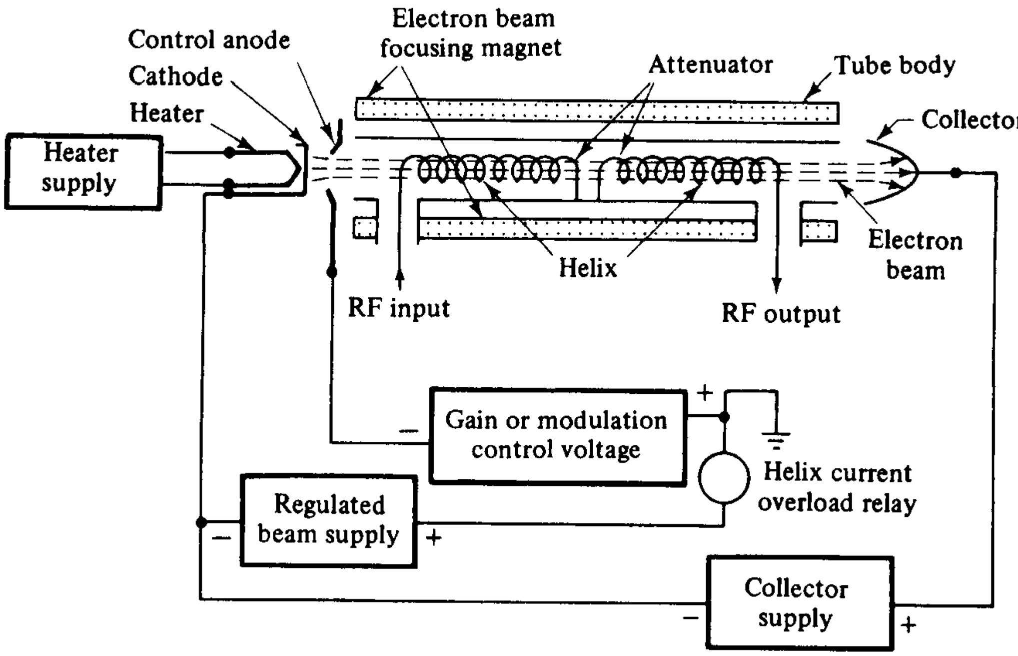


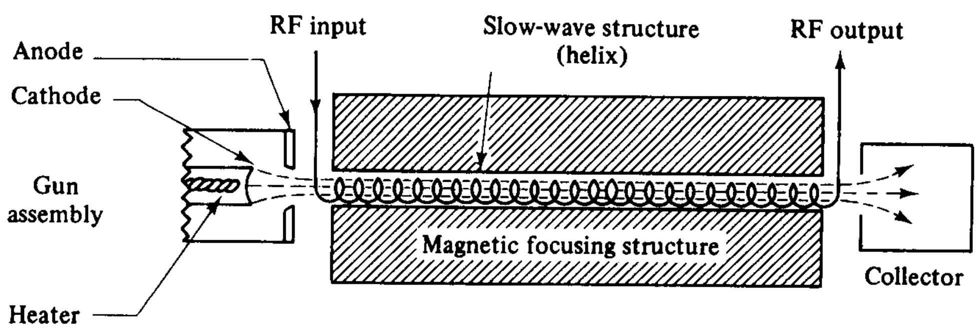
Fig 6.1: Cutaway view of a TWT.

Description

The device is an elongated vacuum tube with an electron gun (a heated cathode that emits electrons) at one end. A magnetic containment field around the tube focuses the electrons into a beam, which then passes down the middle of a wire helix that stretches from the RF input to the RF output, the electron beam finally striking a collector at the other end. The applied RF signal propagates around the turns of the helix and produces and electric field at the center of the helix, with direction of propagation along helix axis.

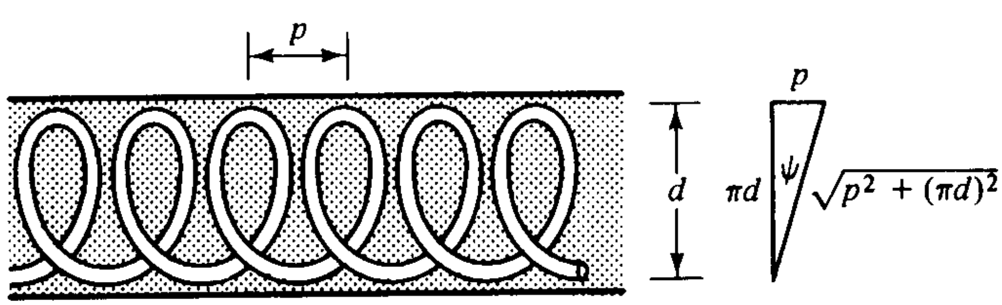


(a)



(b)

Fig 6.2: Helix Traveling Wave Tube (a) Schematic Diagram (b) Simplified Circuit



(a) (b)

Fig 6.3: Helical Slow Wave Structure (a) Helical Coil (b) One turn of helix

This is termed as O-type traveling wave tube. The slow-wave structure is either the helical type or folded back type. The applied signal propagates around the turns of the helix and produces an electric field at the center of the helix, directed along the helix axis. The helix acts as a delay line, in which the RF signal travels at near the same speed along the tube as the electron beam. The axial electric field progresses with a velocity that is very close to the velocity of light multiplied by the ratio of helix pitch to helix circumference. When electrons enter the helix tube, an interaction takes place between the moving axial field and the moving electrons. On the average, the electrons transfer energy to the wave on the helix. This interaction causes the signal wave on the helix to become larger. The electrons entering the helix at zero fields are not affected by the signal wave; those electrons entering the helix at the accelerating field are accelerated, and those at the retarding field are decelerated. As the electrons travel further along the helix, they bunch at the collector end. The bunching shifts the phase by $\pi/2$. Each electron in the bunch encounters a stronger retarding field. Then the microwave energy of the electrons is delivered by the electrons bunch to the wave on helix. The amplification of the signal wave is accomplished.

An attenuator placed on the helix, usually between the input and output helices, prevents reflected wave from traveling back to the cathode and there by suppresses the oscillations if any.

Higher powered TWT's usually contain beryllium oxide ceramic as both a helix support rod and in some cases, as an electron collector for the TWT because of its special electrical, mechanical, and thermal properties.

Slow wave structures

Slow-wave structures are special circuits that are used in microwave tubes to reduce the wave velocity in a certain directions so that the electron beam and the signal wave can interact. The phase velocity of a wave in ordinary waveguides is greater that the velocity of light in a vacuum. In the operation of traveling wave and magnetron type devices, the electron beam must keep in step with the microwave signal. Since the electron beam can be accelerated only to velocities that are about a fraction of the velocity of light, a slow wave structure must be incorporated in the microwave devices so that the phase velocity of the microwave signal can keep pace with that of the electron beam for effective interactions. Several types of slow-wave structures are shown in the figure given below.

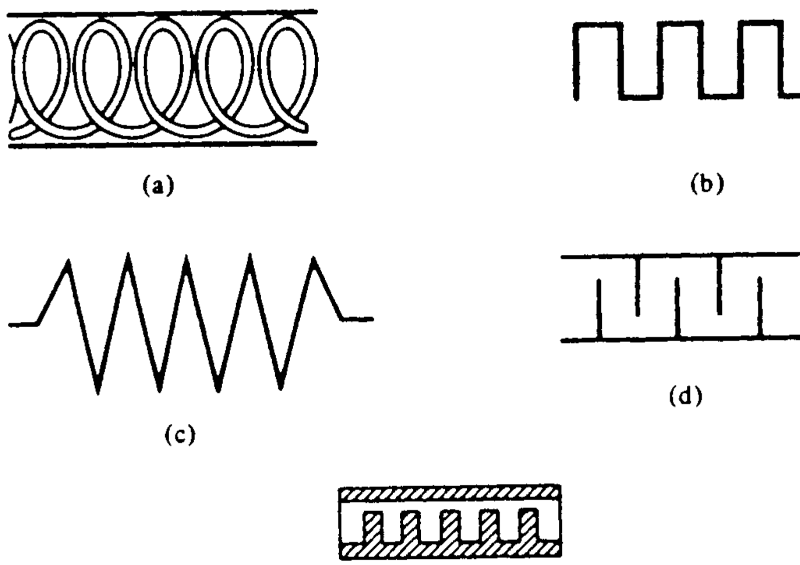


Fig 6.4: Slow wave structures (a) Helical line (b) Folded-back line (c) Zigzag line (d) Inter-digital line (e) Corrugated waveguide.

Traveling-wave tube amplifier

v_p = phase velocity of the em wave along the axial direction = ω / β

p = helix pitch

d = diameter of the pitch

ψ = pitch angle

β = phase constant along the axis.

ω = angular frequency of the wave.

v_g = Group velocity of the em wave.

β_o = phase constant of dc electron beam along the axis.

β_e = phase constant of the velocity modulated electron beam along the axis.

v_o = velocity of dc electron beam = $[2eV_o/m]^{1/2}$

l = length of the slow wave structure.

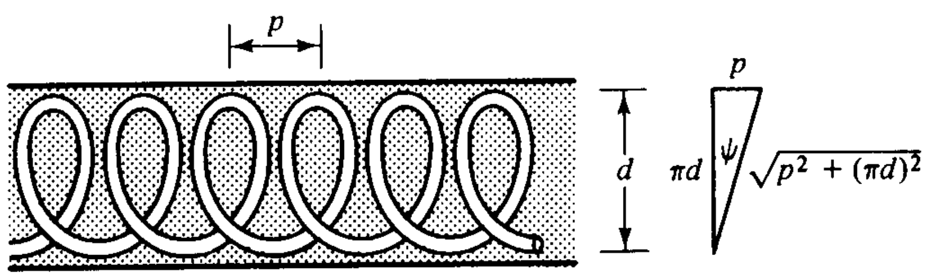
N = $\frac{l}{\lambda_e}$

Applications

1. TWTAs are commonly used as amplifiers in satellite transponders, where the input signal is very weak and the output needs to be high power.
2. TWT is used as transmitter amplifier particularly in airborne and shop borne fire-control radar systems, Satellites, and in electronic warfare and self-protection systems. In these types of applications, a control grid is typically introduced between the TWT's electron gun and slow-wave structure to allow pulsed operation. The circuit that drives the control grid is usually referred to as a grid modulator.

Another major use of TWTAs is for the electromagnetic compatibility (EMC) testing industry for immunity testing of electronic devices.

Mathematical Analysis



The phase velocity 'V_p 'of em wave in the axial direction is given by

$$V_p = c\,\frac{p}{\sqrt{p^2+(\pi d)^2}} = c.\sin\psi \qquad (6.1)$$

If the helix is filled with dielectric material, then

$$V_p = c\,\frac{p}{\sqrt{\mu\epsilon}\sqrt{p^2+(\pi d)^2}} \qquad (6.2)$$

For small pitch angle, p << d, the eqn 6.1 becomes

$$V_p = c\,\frac{p}{\pi d} \qquad (6.3)$$

If λ_a is the wavelength along the axial direction

$$V_p = \lambda_a . f = \frac{2\pi\lambda a\, f}{2\pi} = 2\pi f\,\frac{\lambda a}{2\pi} = \frac{\omega}{\beta} \qquad (6.4)$$

The group velocity V_g is given by

$$V_g = \frac{\partial\omega}{\partial\beta} \qquad (6.5)$$

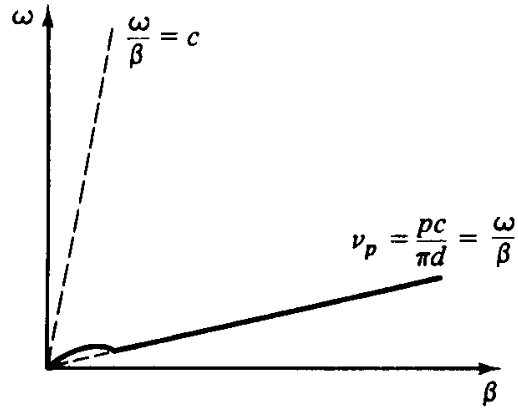


Fig 6. 5: Brillouin Digram of Helical structure

Fig 6.5 shows the ω-β or Brillouin diagrm for a helical slow wave structure. The diagram is very useful in designing a helix slow wave structure. Once β is found, v_p can be computed from eqn 6.4 for a given dimension of the helix. The group velocity of the wave is merely the slope of the curve given by eqn 6.5.

The motion of electrons in the helix type TWT can be quatitatively analyzed interms of the axial electric field. If the travelling wave is propagating in the z-direction.

E_z = z-component of electric field = $E_1 \sin(\omega t - \beta_p z)$ (6.6)

Where β_p = axial phase constant of em wave

$\beta_p = \frac{\omega}{V_p}$

The equation of motion of electron is given by

$$m \frac{dv}{dt} = -e E_1 \sin(\omega t - \beta_p z) \quad (6.7)$$

The electron velocity is given by dc + ac components

$$v = v_0 + v_e \cos(\omega_e t + \theta_e) \quad (6.8)$$

Where

v_0 = *dc elctron velocity*

v_e = magnitude of velocity fluctuation

ω_e = Angular frequency of velocity fluctuations

θ_e = Phase angle of fluctuations

$$\frac{dv}{dt} = -\, v_e\, \omega_e \sin(\omega_e t + \theta_e) \qquad (6.9)$$

Substituting eqn 6.9 in eqn 6.7

$$m\, v_e\, \omega_e \sin(\omega_e t + \theta_e) = e\, E_1 \sin(\omega t - \beta_p z) \qquad (6.10)$$

$$z = \text{distance travelled by the electrons} = v_o\,(t - t_0) \qquad (6.11)$$

Substituting eqn 6.11 in eqn 6.10

$$m\, v_e\, \omega_e \sin(\omega_e t + \theta_e) = e\, E_1 \sin\{\omega t - \beta_p\, v_o\,(t - t_0)\} \qquad (6.12)$$

$$m\, v_e\, \omega_e \sin(\omega_e t + \theta_e) = e\, E_1 \sin\{(\omega - \beta_p v_o)t - \beta_p v_o\, t_0)\} \qquad (6.13)$$

Comparing LHS and RHS terms of above equation we can write

$$v_e = \frac{eE_1}{m\omega_e} \qquad (6.14)$$

$$\omega_e = \beta_p(v_p - v_o) \qquad (6.15)$$

$$\theta_e = \beta_p v_o\, t_0 \qquad (6.16)$$

It can be seen from equation 6.14 that the magnitude of velocity fluctuation of the electron beam is directly proportional to the magnitude of the axial electric field.

Convection current

The convection current 'i' induced in the electron beam by axial electric field is given by (Derivation not in the purview of the syllabus)

$$i = j\, \frac{\beta e\, I_0 E_1}{2\, V_0 (j\beta e - \gamma^2)} \qquad (6.17)$$

Where I_o = dc current

V_o = dc voltage

Where $\beta_e = \frac{\omega}{v_0}$, $v_0 = \sqrt{\frac{2eV_0}{m}}$

and $\gamma = \alpha_e + j\,\beta_e$ propagation constant of axial wave

Axial Field

Equivalent circuit of slow wave structure is given by

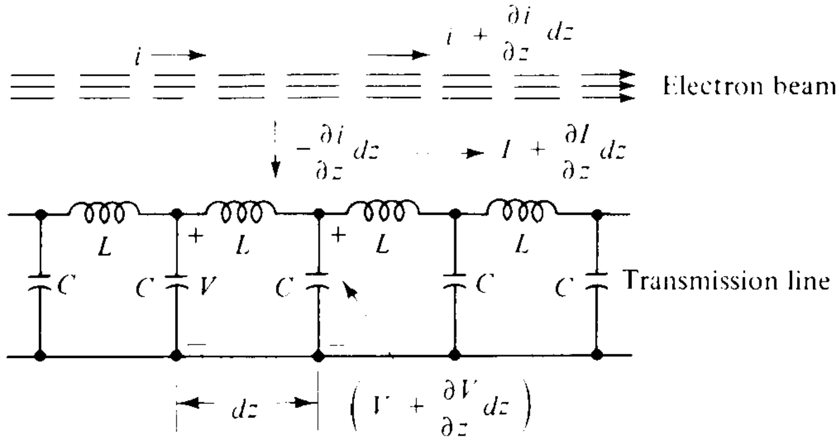


Fig 6.6: electron beam coupled to equivalent circuit of slow wave structure

Where L = inductance per unit length

C = capacitance per unit length

I = alternating current in transmission line

V = alternating voltage in transmission line

Z0 = Characteristic Impedance = $\frac{\sqrt{L}}{\sqrt{C}}$

The wave equation of transmission line is given by

$\gamma^2 V = - V \omega^2 LC - \gamma\, i\, j\, \omega L$ (6.18)

$\gamma = \gamma_o$ when i = 0, $\gamma = \frac{\partial}{\partial z}$

$\gamma_o = j\omega \sqrt{LC}$ (6.19)

$V = \frac{\gamma\gamma_o Z_0}{(\gamma^2 - \gamma_o^2)}\, i$ (6.20)

E_z = Axial field = $- \Delta V = -\frac{\partial V}{\partial z} = - \gamma z$

$E_z = - \frac{\gamma^2\gamma_o Z_0}{(\gamma^2 - \gamma_o^2)}\, i$ (6.21)

Wave modes and propagation constants

The wave modes of helix type TWT can be determined by solving equations 6.16 and 6.21 simultaneously for propagation constants.

Each solution for the propagation constant 'γ' represents a mode of travelling wave in the tube. There are four distinct solution for the propagation constants. This means that there are four modes of travelling waves which are given by:

$$\gamma_1 = -\beta_e c \frac{\sqrt{3}}{2} + j\beta_e \left(1+\frac{c}{2}\right) \quad (6.22)$$

$$\gamma_2 = \beta_e c \frac{\sqrt{3}}{2} + j\beta_e \left(1+\frac{c}{2}\right) \quad (6.23)$$

$$\gamma_3 = j\beta_e (1-c) \quad (6.24)$$

$$\gamma_4 = -j\beta_e \left(1-\frac{c^3}{4}\right) \quad (6.25)$$

Examining the real and imaginary parts of propagating constant we can understand that

1. Wave corresponding to γ_1 is a **forward wave** and its **amplitude increases exponentially** with distance.
2. Wave corresponding to γ_2 is also a **forward wave** but its **amplitude decays exponentially** with distance.
3. Wave corresponding to γ_3 is also a **forward wave** but its **amplitude remains constant** with distance.
4. Wave corresponding to γ_3 is a **backward wave** but its **amplitude remains constant** with distance.

Output voltage, gain and efficiency

Let V_{in} = amplitude of input voltage when z = 0

Let V_{out} = amplitude of output voltage when z= ℓ

The expressions for Voltage output, gain parameter and power gain are given by

$$V_{out} = \frac{V_{in}}{3} e^{\sqrt{3}\,N\,C'} \quad (6.26)$$

Where c' = Gain Parameter $=\left[\frac{I_o Z_o}{V_o}\right]^{\frac{1}{3}}$

Power gain = $A_p = 10 \log_{10}\left[\frac{V_{out}}{V_{in}}\right]^2 = -9.54 + 47.3\,N\,C'$ dB

Where $N = \frac{L}{\lambda_e}$ and $\lambda_e = \frac{2\pi}{\beta_e}$ Where $\beta_e = \frac{\omega}{v_o}$

Eg1: The parameters of a TWT are beam voltage V_o = 2.5 kV, beam current I_o = 25 mA, characteristic impedance Z_o = 10 Ω and operating frequency f= 9.5 GHz. Circuit length N = 40, Calculate

(a) *Gain parameter C'*
(b) *Output power gain A_p*
(c) *Electronic Phase Constant β_e*

Solution

c' = Gain Parameter $=[\frac{I_o Z_o}{V_o}]^{\frac{1}{3}} = [\frac{25\ X\ 10^{-3} X\ 10}{2.5 X\ 10^3}]^{\frac{1}{3}}$ = 0.0464

Power gain A_p **=** - 9.54 + (47.3 X 40 X 0.0464) = 78.2 dB

$v_0 = 0.593 \text{ x } 10^6 \sqrt{Vo} = 0.593 \text{ x } 10^6 \sqrt{2500} = 2.965 \text{ X } 10^7$ m/s

Electronic Phase Constant $\beta_e = \frac{\omega}{v_0} = \frac{2\pi\ X\ 9.5\ X 10^9}{2.965\ X 10^7}$ = *2.01 X10³ rad/m*

Eg2: A helix TWT operates at 4 GHz under a beam voltage of 10 kV and beam current of 500 mA, helix impedance is 25 Ω and interaction length is 20 cm. Find the output power gain.

$v_0 = 0.593 \text{ x } 10^6 \sqrt{Vo} = 0.593 \text{ x } 10^6 \sqrt{10000} = 0.593 \text{ X } 10^8$ m/s

$\beta_e = \frac{\omega}{v_0} = \frac{2\pi\ X\ 4\ X 10^9}{0.593\ X 10^8} = \frac{2\pi}{\lambda e}$

$\lambda_e = \frac{0.593\ X 10^8}{4\ X 10^9}$ = *0.014825 m*

$N = \frac{L}{\boldsymbol{\lambda e}} = \frac{20\ X 10^{-2}}{0.014825}$ = 13.49

c' $= [\frac{I_o Z_o}{V_o}]^{\frac{1}{3}} = [\frac{500\ X\ 10^{-3} X\ 25}{4 X 10^4}]^{\frac{1}{3}}$ = 0.068

Power gain A_p **=** - 9.54 + (47.3 X 13.49 X 0.068) = 33.85 dB

Eg3: (May 2010) A TWT operates with following parameters. V_o = 2.5kV, I_o= 25 mA, Z_o = 10 Ω , L = 50, f = 9 GHz. Find out Gain parameter and power gain.

Solution

$v_0 = 0.593 \text{ x } 10^6 \sqrt{Vo} = 0.593 \text{ x } 10^6 \sqrt{2500} = 0.296 \text{ X } 10^8$ m/s c' = $[\frac{I_o Z_o}{V_o}]^{\frac{1}{3}} = [\frac{25\ X\ 10^{-3} X\ 10}{2.5 X 10^3}]^{\frac{1}{3}}$ = 0.0464

Power gain A_p **=** - 9.54 + (47.3 X 50 X 0.0464) = 100.2 dB

MAGNETRON

Figure 6.6: Magnetron used in Russian Radar

Historical background

In 1921 **Albert Wallace Hull** invented the magnetron as a microwave tube. During World War II it was developed by John Randall and Henry Boot to a powerful microwave generator for Radar applications.

Magnetrons function as self-excited microwave oscillators. Crossed electron and magnetic fields are used in the magnetron to produce the high-power output required in radar equipment. These multi-cavity devices may be used in radar transmitters as either pulsed or CW oscillators at frequencies ranging from approximately 0.6 to 30 G Hz. The relatively simple construction has the disadvantage that the Magnetron usually can work only on a constructively fixed frequency.

Physical construction of a magnetron

The magnetron is classed as a diode because it has no grid. The anode of a magnetron is fabricated into a cylindrical solid copper block. The cathode and filament are at the center of the tube and are supported by the filament leads. The filament leads are large and rigid enough to keep the cathode and filament structure fixed in position. The cathode is indirectly heated and is constructed of a high-emission material. The 8 up to 20 cylindrical holes around its circumference are resonant cavities. The cavities control the output frequency. A narrow slot runs from each cavity into the central portion of the tube dividing the inner structure into as many segments as there are cavities.

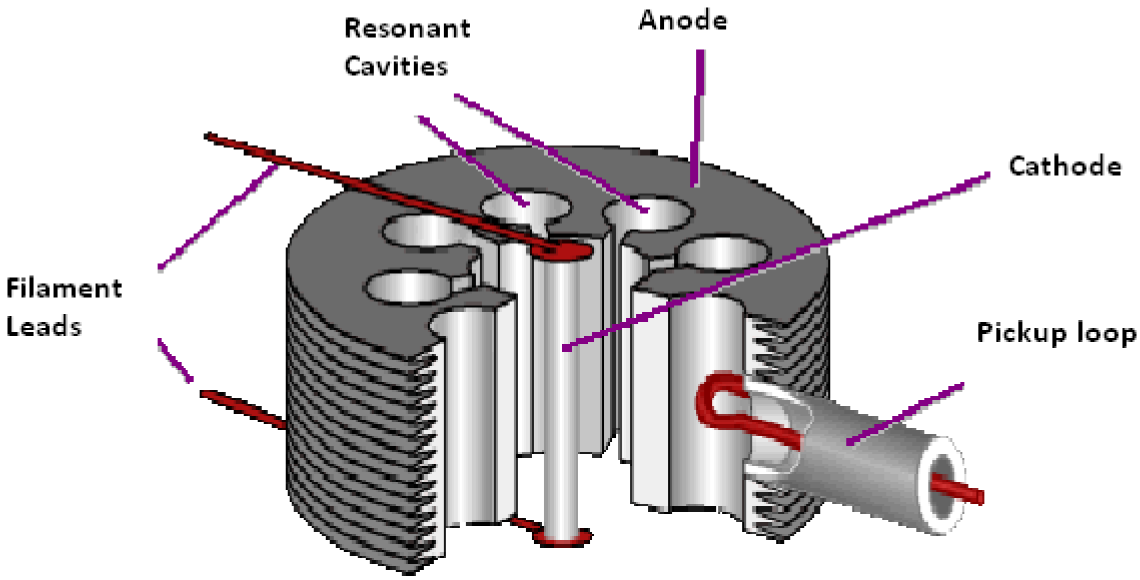


Figure 6.7: Cutaway view of a magnetron

The open space between the plate and the cathode is called the interaction space. In this space the electric and magnetic fields interact to exert force upon the electrons. The magnetic field is usually provided by a strong, permanent magnet mounted around the magnetron so that the magnetic field is parallel with the axis of the cathode and is perpendicular to electron beam.

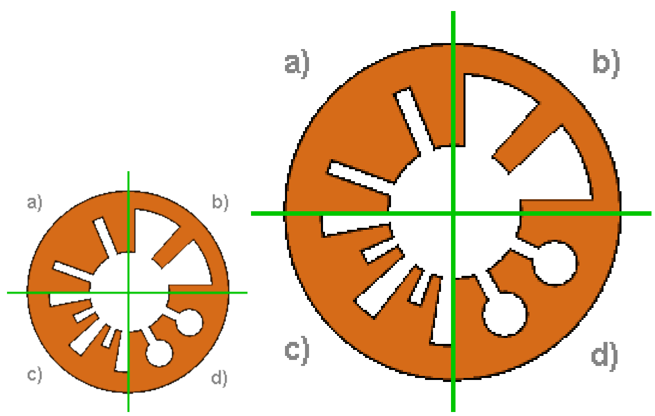


Figure 6.8: forms of the plate(Resonant Cavities) of magnetrons

The form of the cavities varies, shown in the Figure 6.8. The output lead is usually a probe or loop extending into one of the tuned cavities and coupled into a waveguide or coaxial line.

a. **slot- type**
b. **vane- type**
c. **rising sun- type**
d. **hole-and-slot- type**

Basic Magnetron Operation

As when all velocity-modulated tubes the electronic events at the production microwave frequencies at a Magnetron can be subdivided into four phases too:

1. phase: Production and acceleration of an electron beam
2. phase: Velocity-modulation of the electron beam
3. phase: Forming of a „Space-Charge Wheel"
4. phase: Dispense energy to the ac field

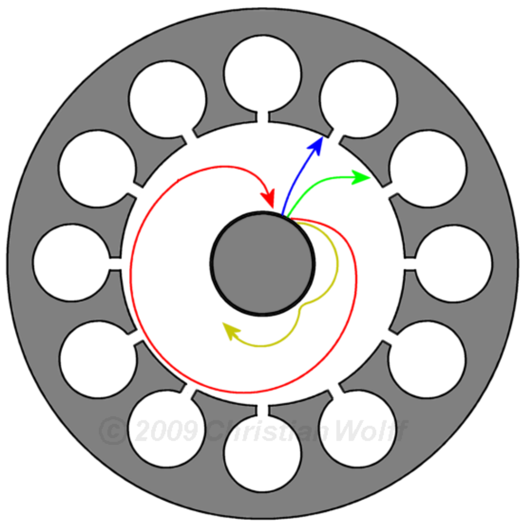


Figure 6.9: the electron path under the influence of different strength of the magnetic field

1. Phase: Production and acceleration of an electron beam

When no magnetic field exists, heating the cathode results in a uniform and direct movement of the field from the cathode to the plate (the blue path in figure 6.9). The permanent magnetic field bends the electron path. If the electron flow reaches the plate, so a large amount of plate current is flowing. If the strength of the magnetic field is increased, the path of the electron will have a sharper bend. Likewise, if the velocity of the electron increases, the field around it increases and the path will bend more sharply. However, when the critical field value is reached, as shown in the figure as a red path, the electrons are deflected away from the plate and the plate current then drops quickly to a very small value. When the field strength is made still greater, the plate current drops to zero.

When the magnetron is adjusted to the cutoff, or critical value of the plate current, and the electrons just fail to reach the plate in their circular motion, it can produce oscillations at microwave frequencies.

2. Phase: Velocity-modulation of the electron beam

The electric field in the magnetron oscillator is a product of ac and dc fields. The dc field extends radially from adjacent anode segments to the cathode. The ac fields, extending between adjacent segments, are shown at an instant of maximum magnitude of one alternation of the RF oscillations occurring in the cavities.

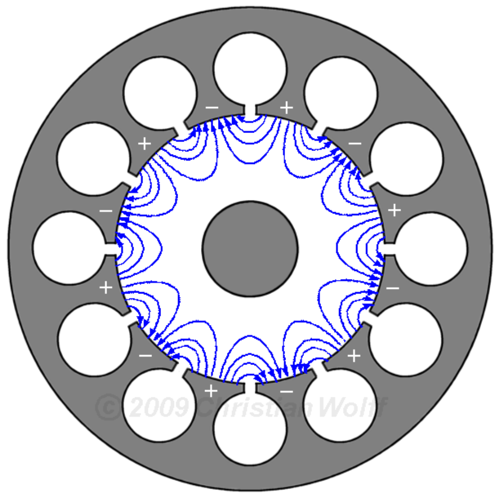


Figure 6.10: The high-frequency electrical field

In the figure 6.10 is shown only the assumed high-frequency electrical ac field. This ac field work in addition to the to the permanently available dc field. The ac field of each individual cavity increases or decreases the dc field like shown in the figure.

Well, the electrons which fly toward the anode segments loaded at the moment more positively are accelerated in addition. These get a higher tangential speed. On the other hand the electrons which fly toward the segments loaded at the moment more negatively are slow down. These get consequently a smaller tangential speed.

3. Phase: Forming of a "Space-Charge Wheel"

On reason the different speeds of the electron groups a velocity modulation appears therefore.

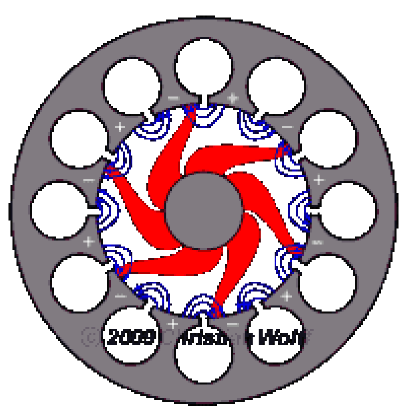


Figure 6.11: Rotating space-charge wheel in an 12-cavity magnetron

The cumulative action of many electrons returning to the cathode while others are moving toward the anode forms a pattern resembling the moving spokes of a wheel known as a “Space-Charge Wheel”, as indicated in figure 6.11. The space-charge wheel rotates about the cathode at an angular velocity of 2 poles (anode segments) per cycle of the ac field. This phase relationship enables the concentration of electrons to continuously deliver energy to sustain the RF oscillations.

One of the spokes just is near an anode segment which is loaded a little more negatively. The electrons are slowed down and pass her energy on to the ac field. This state isn't static, because both the ac- field and the wire wheel permanently circulate. The tangential speed of the electron spokes and the cycle speed of the wave must be brought in agreement so.

4. Phase: Dispense energy to the ac field

Figure 7: Path of a single electron under influence of the electric RF-field

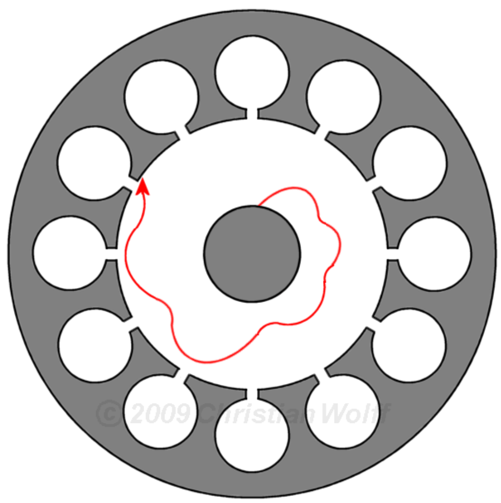


Figure 6.12: Path of a single electron under influence of the electric RF-field

Recall that an electron moving against an E field is accelerated by the field and takes energy from the field. Also, an electron dispense energy to a field and slows down if it is moving in the same direction as the field (positive to negative). The electron spends energy to each cavity as it passes and eventually reaches the anode when its energy is expended. Thus, the electron has helped sustain oscillations because it has taken energy from the dc field and given it to the ac field. This electron describes the path shown in figure 6.12 over a longer time period looked. By the multiple breaking of the electron the energy of the electron is used optimally. The effectiveness reaches values up to 80%.

Modes of Oscillation

The operation frequency depends on the sizes of the cavities and the interaction space between anode and cathode. But the single cavities are coupled over the interaction space with each other. Therefore several resonant frequencies exist for the complete system. Two of the four possible waveforms of a magnetron with 8 cavities are in the figure 6.13 represented. Several other modes of oscillation are possible (3/4π, 1/2π, 1/4π), but a magnetron operating in the π mode has greater power and output and is the most commonly used.

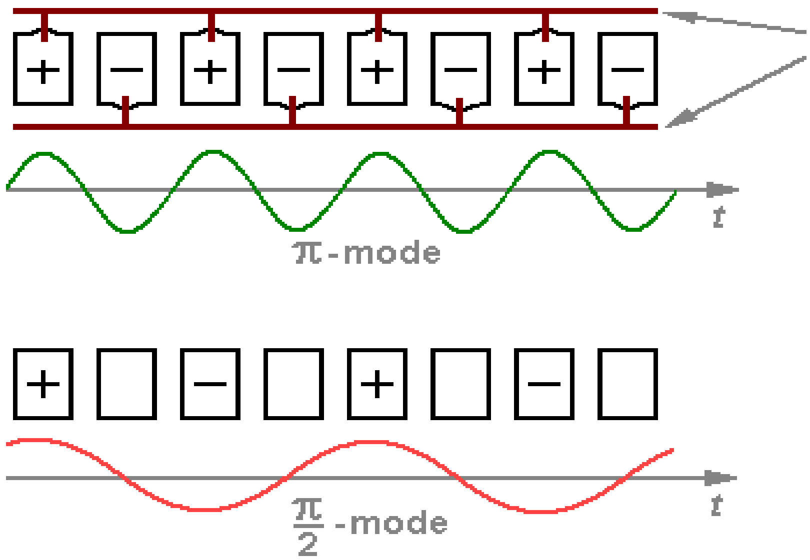


Figure 6.13: Waveforms of the magnetron
(Anode segments are represented „unwound”)

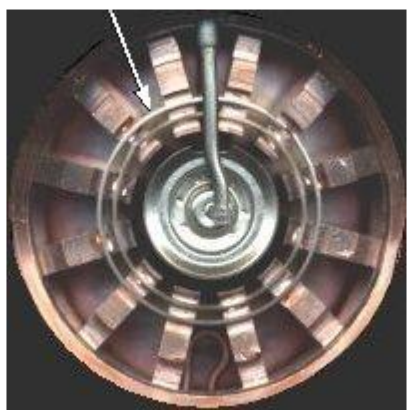


Figure 6.14: Strapping: cutaway view of a magnetron (vane-type), showing the strapping rings and the slots.

So that a stable operational condition adapts in the optimal pi mode, two constructive measures are possible:

- **Strapping rings:**
  The frequency of the π mode is separated from the frequency of the other

modes by strapping to ensure that the alternate segments have identical polarities. For the pi mode, all parts of each strapping ring are at the same potential; but the two rings have alternately opposing potentials. For other modes, however, a phase difference exists between the successive segments connected to a given strapping ring which causes current to flow in the straps.

- Use of cavities of **different resonance frequency**
  E.g. such a variant is the anode form „Rising Sun”.

Magnetron coupling methods

Energy (RF) can be removed from a magnetron by means of a coupling loop. At frequencies lower than 10,000 megahertz, the coupling loop is made by bending the inner conductor of a coaxial line into a loop. The loop is then soldered to the end of the outer conductor so that it projects into the cavity, as shown in figure 6.15, view (A). Locating the loop at the end of the cavity, as shown in view (B), causes the magnetron to obtain sufficient pickup at higher frequencies.

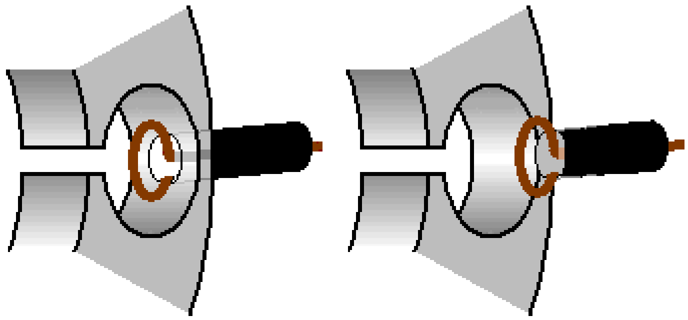


Figure 6.15: Magnetron coupling, view (A) and (B)

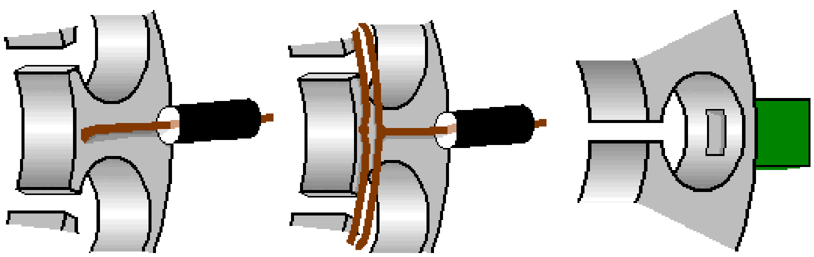


Figure 6.15: Magnetron coupling, view (C), (D) and (E)

The segment-fed loop method is shown in view (C) of figure 6.15. The loop intercepts the magnetic lines passing between cavities. The strap-fed loop method (view (D), intercepts the energy between the strap and the segment. On the output side, the coaxial line feeds another coaxial line directly or feeds a waveguide through a choke joint. The vacuum seal at the inner conductor helps to support the line. Aperture, or slot, coupling is illustrated in view (E). Energy is coupled directly to a waveguide through an iris.

Magnetron tuning

A tunable magnetron permits the system to be operated at a precise frequency anywhere within a band of frequencies, as determined by magnetron characteristics. The resonant frequency of a magnetron may be changed by varying the inductance or capacitance of the resonant cavities.

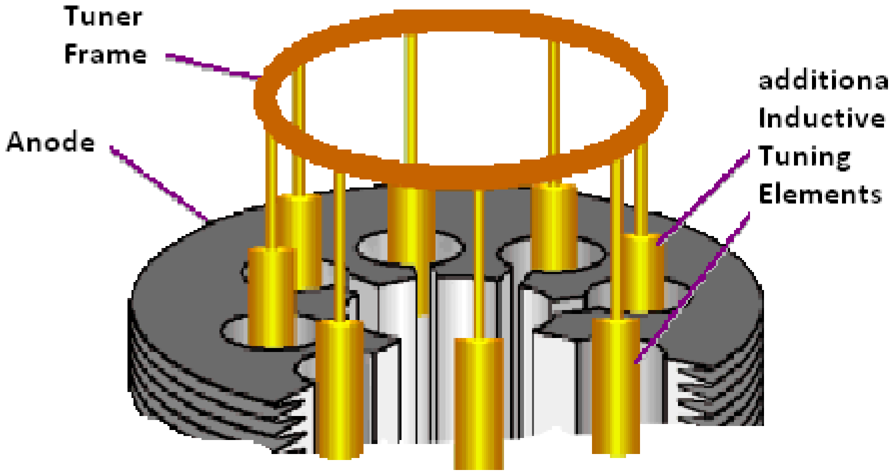


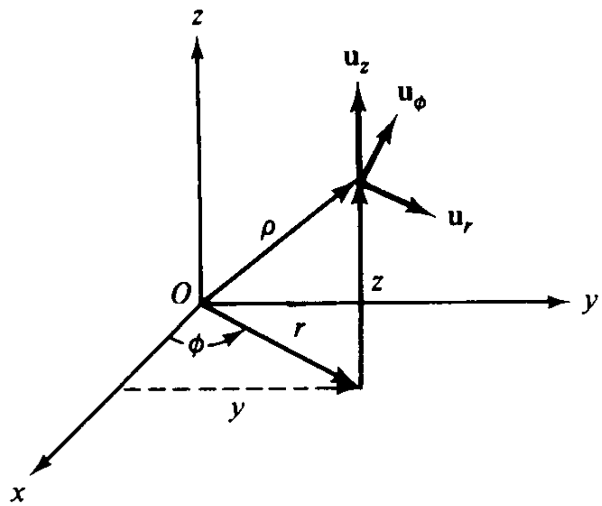
Figure 616: Inductive magnetron tuning

Figure 6.17: resonant cavities of an hole-and-slot- type magnetron with inductive tuning elements

Figure 6.18: Magnetron VMX1090 of the ATC-radar [PAR-80] This magnetron is even equipped with the permanent magnets necessary for the work.

Mathematical Analysis of Magnetron

Since the magnetron is of cylindrical shape we use cylindrical coordinates as shown below for our mathematical analysis.



Where x= r cosΦ

y= r sinΦ and z = z

In a magnetron the electron is subjected to a force due to electric field and magnetic field. The expression for the force is

$$\mathbf{F} = -e\mathbf{E} - (e\mathbf{v} \times \mathbf{B}) \quad (6.27)$$

The equation of motion of electron is given by

$$F = m\frac{dv}{dt} = m\sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2} \text{ in rectangular coordinates.} \quad (6.28)$$

Veclocity in cylindrical coordinaes is expressed by

$$\mathbf{v} = \frac{dr}{dt}\,\boldsymbol{v_r} + r\frac{d\Phi}{dt}\,\boldsymbol{v}_\Phi \quad \text{neglecting } v_z \quad (6.29)$$

Where $\boldsymbol{v_r}$ and $\boldsymbol{v}_\Phi$ are unit vectors

$$\frac{dv}{dt} = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\Phi}{dt}\right)^2\right]\boldsymbol{v_r} + \left[r\frac{d^2\Phi}{dt^2} + 2\,\frac{dr}{dt}\,\frac{d\Phi}{dt}\right]\boldsymbol{v}_\Phi \quad (6.30)$$

$$= \left[\frac{d^2r}{dt^2} - r\left(\frac{d\Phi}{dt}\right)^2\right]\boldsymbol{v_r} + \frac{1}{r}\frac{d}{dt}\left[r^2\,\frac{d\Phi}{dt}\right]\boldsymbol{v}_\Phi \quad (6.31)$$

The term v X B in cylindrical coordinates is given by (assuming B exists in only z direction.

$$\mathbf{v} \times \mathbf{B} = (B_z r \frac{d\Phi}{dt})\, \boldsymbol{v_r} - (B_z \frac{dr}{dt})\, \boldsymbol{v_\Phi} \qquad (6.31)$$

Equating the vr and vΦ terms from above equations, we can write

$$\frac{d^2r}{dt^2} - r\left(\frac{d\Phi}{dt}\right)^2 = \frac{e}{m} E_r - \frac{e}{m} r B_z \frac{d\Phi}{dt} \qquad (6.34)$$

$$\frac{1}{r}\frac{d}{dt}\left[r^2 \frac{d\Phi}{dt}\right] = \frac{e}{m} \text{ r Bz } \frac{dr}{dt} = \frac{1}{2}\frac{e}{m} \text{ Bz } \frac{d}{dt}(r^2) = \frac{1}{2}\,\omega_c\, \frac{d}{dt}(r^2) \qquad (6.35)$$

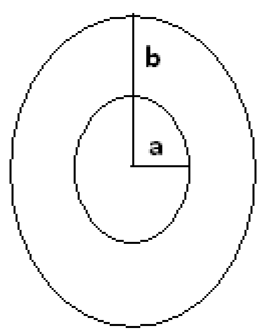
Where $\omega_c = \frac{e}{m}$ Bz Cyclotron angular frequency

Integrating equation 6.35

$$r^2 \frac{d\Phi}{dt} = \frac{1}{2}\,\omega_c\, r^2 + \text{Constant} \qquad (6.36)$$

Let a = radius of cathode of magnetron

b = radius of anode



At r = a , $\frac{d\Phi}{dt}$ = 0, therefore the constant in equation 6.36 becomes

Constant = $-\frac{1}{2}\,\omega_c\, a^2$

$$\frac{d\Phi}{dt} = \frac{1}{2}\,\omega_c \left(1 - \frac{a^2}{r^2}\right) \qquad (6.37)$$

Since the magnetic field does no work on electrons, the kinetic energy of electrons is given be

$$\frac{1}{2}\, mv^2 = \text{e V} \qquad (6.38)$$

Where $v^2 = (\frac{dr}{dt})^2 + (r\frac{d\Phi}{dt})^2$ (6.39)

At r = b, V = V_o, $\frac{dr}{dt} = 0$ (6.40)

From Eqn 6.37

$$\frac{d\Phi}{dt} = \frac{1}{2}\,\omega_c\,(1 - \frac{a^2}{b^2}) \qquad (6.41)$$

From equations 6.38 and 6.39

$$v^2 = (\frac{dr}{dt})^2 + (r\frac{d\Phi}{dt})^2 = \frac{2eV}{m}$$

At r = b, $\frac{dr}{dt} = 0$, therefore $(b\frac{d\Phi}{dt})^2 = \frac{2eV_o}{m}$ (6.42)

Substituting for $\frac{d\Phi}{dt}$ from equation 6.41

$$b^2[\frac{1}{2}\,\omega_c\,(1 - \frac{a^2}{b^2})]^2 = \frac{2eV_o}{m}$$

$$\therefore \omega_c{}^2 = \frac{\frac{8eV_o}{m}}{b^2}\,[\frac{1}{(1-\frac{a^2}{b^2})}]^2 = [\frac{e}{m}\,\text{Bz}]^2$$

$$\therefore B_z = \frac{\sqrt{\frac{8mV_o}{e}}}{b\,(1-\frac{a^2}{b^2})} \qquad (6.43)$$

Hull cut-0ff magnetic field **'B_{oc}'** is the magnetic field value above which the anode current becomes zero for given V_o

$$\boldsymbol{B_{oc}} = \frac{\sqrt{\frac{8mV_o}{e}}}{b(1-\frac{a^2}{b^2})} \qquad (6.44)$$

Similarly for given B_z of B_o the anode current becomes zero for $V_o < V_{oc}$

From equation 6.43, we can write the expression for V_{oc}

$$\mathbf{V_{oc}} = \frac{e}{8m}\mathbf{B_o}^2\, \mathbf{b}^2(1- \frac{a^2}{b^2})^2 \qquad (6.45)$$

<u>Hartree Condition:</u> The hull cutoff condition determines the anode voltage or magnetic field necessary to obtain non-zero anode current as a function of magnetic field or anode voltage in the absence of electromagnetic field.

<u>Frequency Pushing</u>: It is the variation of the frequency of the magnetron due to the changes in anode voltage. The change in oscillator frequency produced by a change in the mode current for a fixed load is called pushing figure. A plot frequency versus current is called pushing characteristic and the slope of this curve represents the pushing figure. To avoid frequency pushing stabilized anode voltage power supply need to be used.

<u>Frequency Pulling</u>: It is the variation of the frequency of the magnetron due to the changes in load impedance. The change in oscillator frequency produced by a change in the mode current for a fixed load is called pushing figure. A plot frequency versus the load is called pulling characteristic and the slope of this curve represents the pulling figure. To avoid frequency pulling load should be stable.

Eg1: A normal cylindrical magnetron has the following parameters

Inner radius = 0.15 m

Outer radius = 0.45 m

Bo = 1.2 milli Webers /m^2

Findout (a) Hull cutoff Voltage (b) Cutoff magnetic field for V_o = 6 kV

<u>Solution</u>

e/m = 1.759 X 10^{11}

(a) $V_{oc} = \frac{e}{8m} B_o^2\, b^2 (1- \frac{a^2}{b^2})^2$ = 5.04 kV

(b) $B_{oc} = \frac{\sqrt{\frac{8mV_o}{e}}}{b(1-\frac{a^2}{b^2})}$ = 1.3 m Web / m^2

Eg 2: A Magnetron operates with following parameters (May 2009)
Vo=25 kV, Io=25 A, Bo=0.34 T, Diameter of cathode =8 cm,
Radius of vane edge to centre= 8 cm.

Find the cyclotron frequency and cut off voltage.

a= 4 cm, b= 8 cm, 1 Tesla = 1 Web/m^2

$\omega_c = \frac{e}{m}$ Bz = 1.759 X 10^{11} x 0.34 = 5.9808 X 10^{10} Rad /s

$\mathbf{V_{oc} = \frac{e}{8m} B_o^2\, b^2 \left(1 - \frac{a^2}{b^2}\right)^2}$ = 9150 kV