UNIT-VII

DETECTION OF READAR SIGNALS IN NOISE

7.1 Matched Filter

In this unit some aspects of the problem of detecting radar signals in the presence of noise will be considered. Noise ultimately limits the capability of any radar. The detection of signals in the presence of clutter is always a challenging task for the design engineers.

A network whose frequency-response function maximizes the output peak-signal-to-mean noise (power) ratio is called a *matched filter*. This criterion, or its equivalent, is used for the design of almost all radar receivers.

The frequency-response function, denoted **H(f)**, expresses the relative amplitude and phase of the output of a network with respect to the input when the input is a pure sinusoid.

The magnitude I **H(**f)I of the frequency-response function is the receiver amplitude pass band characteristic.

Case I: If the bandwidth of the receiver pass band is wide compared with that occupied by the signal energy, extraneous noise is introduced by the excess bandwidth which lowers the output signal-to-noise ratio.

Case-II: On the other hand, if the receiver bandwidth is narrower than the bandwidth occupied by the signal, the noise energy is reduced along with a considerable part of the signal energy. The net result is again a lowered signal-to-noise ratio.

Thus there is an optimum bandwidth at which the signal-to-noise ratio is a maximum. This is well known to the radar receiver designer. The rule of thumb quoted in pulse radar practice is that the receiver bandwidth **B** should be approximately equal to the reciprocal of the pulse width ***T.***

As we shall see later, this is a reasonable approximation for pulse radars with conventional Super heterodyne receivers. It is not generally valid for other waveforms, however, and is mentioned to illustrate in a qualitative manner the effect of the receiver characteristic on signal-to-noise ratio. The exact specification of the optimum receiver characteristic involves the frequency-response function and the shape of the received waveform.

- The receiver frequency-response function, is assumed to apply from the antenna terminals to the output of the IF amplifier.
- The second detector and video portion of the well-designed radar super heterodyne receiver will have negligible effect on the output signal-to-noise ratio if the receiver is designed as a matched filter.)

- Narrow banding is most conveniently accomplished in the IF. The bandwidths of the RF and mixer stages of the normal super heterodyne receiver are usually large compared with the IF bandwidth.

Therefore the **frequency-response function** of the portion of the receiver included between the antenna terminals to the output of the IF amplifier is taken to be that of the IF amplifier alone.

The IF amplifier may be considered as a filter with gain. The response of this filter as a function of frequency is the property of interest.

7.2. Frequency-response function

For a received waveform S(t) with a given ratio of signal energy E to noise energy ***No*** (or noise power per hertz of bandwidth), the frequency-response function of the linear, time-invariant filter which maximizes the output peak-signal-to-mean-noise (power) ratio is given by

$$\mathbf{H(f) = G_a S^*(f) \exp(-j2\pi f t_1)} \quad \text{..................(7.1)}$$

where $S(f) = \int S(t) \exp(-j2\pi f t_1)\, \boldsymbol{dt}$ = voltage spectrum (Fourier transform) of input signal

$S^*(f)$ = complex conjugate of $S(f) = S(-f)$

t_1 = fixed value of time at which signal is observed to be maximum

$\mathbf{G_a}$= constant equal to maximum filter gain (generally taken to be unity)

The noise that accompanies the signal is assumed to be stationary and to have a uniform Spectrum (white noise). It need not be Gaussian.

If the noise is not white, Eq. (7.1) may be modified and will be discussed later in this section. The filter whose frequency-response function is given by Eqn (7.1) has been called the North filter, the conjugate filter, or more usually the matched filter.

The frequency-response function of the matched filter is the conjugate of the spectrum of the received waveform except for the phase shift $\exp(-j2\pi f t_1)$. This phase shift varies uniformly with frequency. Its effect is to cause a constant time delay. A time delay is necessary in the specification of the filter for reasons of physical realizability since there can be no output from the filter until the signal is applied.

The frequency spectrum of the received signal may be written as an amplitude spectrum $|S(f)|$ and a phase spectrum $\exp[-j\Phi_s(f)]$. The matched filter frequency-response function may similarly be written in terms of its amplitude and phase
spectra $|H(f)|$ and $\exp[-j\Phi m(f)]$.

$$S(f) = |S(f)| \exp[-j\Phi s(f)] \qquad (7.2)$$
$$S^*(f) = |S(f)| \exp[+j\Phi s(f)] \qquad (7.3)$$
$$H(f) = |H(f)| \exp[-j\Phi m(f)] \qquad (7.4)$$

Ignoring the constant $\mathbf{G_a}$, substituting Eqn (7.3) and Eqn(7.4) in Eqn (7.1) for the matched filter may then be written as

$$|H(f)| \exp[-j\,\Phi_m(f)] = |S(f)| \exp[j\,\Phi_s(f) - j2\pi f t_1] \qquad (7.5)$$

Or $| H(f) | = |S(f)|$ (7.6)

and $\Phi_m(f) = -\Phi_s(f) + 2\pi f t_1$ (7.7)

Thus the amplitude spectrum of the matched filter is same as the amplitude spectrum of input signal.

The phase spectrum of the matched filter is the negative of phase spectrum of input signal plus a phase shift proportional to the frequency.

Let h(t) be the **impulse response of the matched filter.** Therefore h(t) is the inverse Fourier transform of H(f). by definition

$$h(t) = \int H(f) \exp[j2\pi ft]\, df \quad (7.8)$$

Substituting for H(f) from eqn (7.1)

$$h(t) = G_a \int S^*(f) \exp\{-j2\pi f(t_1 - t)\}\, df \quad (7.9)$$

since $S^*(f) = S(-f)$

$$h(t) = G_a \int S^*(f) \exp\{j2\pi f\ (t_1 - t]\}\, df \quad (7.10)$$

from the definition of Fourier transform **h(t)** can also be written

$$\mathbf{h(t) = G_a\, s(t_1 - t)} \quad (7.11)$$

It can be seen that the impulse response of matched filter is the image of the received signal run backwards in time starting from fixed time 't_1'

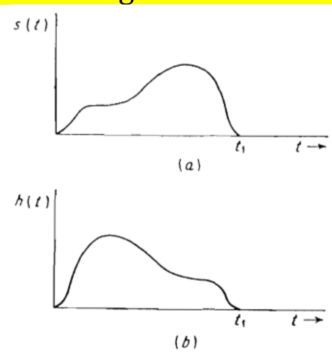

Figure 7.1: (a) Received waveform s (t) ;
(b) impulse response h(t) of the matched filter.

7.3. Derivation of the matched-filter characteristic

The frequency-response function of the matched filter has been derived by a number of authors using either the calculi of variations or the Schwartz inequality. Here we shall derive the matched-filter frequency response function using the Schwartz inequality.

We assume that the input noise is white noise.(Uniform Spectral density)

By definition matched filter is used to maximse

$$R_f = |s_o(t)|^2_{max} / N \qquad (7.12)$$

Where $|s_o(t)|_{max}$ is the maximum value of output signal voltage.

N is the mean power noise at the output.

Let H(f) be the frequency response function of the matched filter. By definition the output signal of matched filter can be written as

$$s_o(t) = |\int S(f)\, H(f) \exp[j2\pi f t_1]\; df| \qquad (7.13)$$

$$N = N_o/2 \int |H(f)^2|\; df \qquad (7.14)$$

Where N_o is the input noise power per unit bandwidth. The factor ½ appears before the integral because of limits extended from $-\infty$ to $+\infty$ where No is defined for positive values only.

Substituting eqns (17.3) and (7.14) in eqn (7.12)

$$R_f = [|\int S(f)\, H(f) \exp[j2\pi f t_1]\; df|]^2 \;/\; [N_o/2 \int |H(f)^2|\; df] \qquad (7.15)$$

Schwartz's inequality states that if P and Q are two complex functions, then

$$\int P^*P\, dx \,.\, \int Q^*Q\, dx \geq |P^*Q\; dx|^2 \qquad (7.16)$$

The equality sign applies when P = kQ, where k is a constant

$$\text{Let } P = S(f) \exp[j2\pi f t_1] \qquad (7.17)$$

$$\text{and } Q = H(f) \qquad (7.18)$$

$$\text{Then } \int P^*P\, dx = \int |P|^2\, dx$$

$$\int |P^*Q\; dx|^2 = |S(f)\, H(f) \exp[j2\pi f t_1]\; df|^2 \qquad (7.19)$$

$$\int P^*P\, dx = \int |S(f)|^2\, df \quad \textit{(omitting the phase)} \qquad (7.20)$$

$$\int Q^*Q\, dx = \int |H(f)|^2\, df \qquad (7.21)$$

$$\int |H(f)|^2\, df \,.\, \int |S(f)|^2\, df \leq |S(f)\, H(f) \exp[j2\pi f t_1]\; df|^2 \qquad (7.22)$$

Dividing both sides of above equation by $N_o/2 \int |H(f)^2|\; df$

$$\frac{\int |S(f)|^2\, df}{N_o/2} \leq R_f \qquad (7.23)$$

From Parseral's Theorem we have

Signal Energy $E = \int |S(f)|^2 \, df = \int |s(t)|^2 \, dt$

Eqn 7.23 can be written as

$$\frac{2E}{N_o} \leq R_f \qquad \textit{(7.24)}$$

The equality sign is applicable if $P = kQ$

$$R_f = \frac{2E}{N_o} \qquad \textit{(7.25)}$$

$$H(f) = k\,S(f) \exp(\,j2\pi f t_1) \qquad (7.26)$$

Where k is constant and assumed to be the gain G_a

Since $S^*(f) = S(-f)$, eqn 26 becomes

$$\mathbf{H(f) = G_a S^*(f) \exp(\,-j2\pi f t_1)} \qquad (7.27)$$

7.4. The matched filter and the correlation function.

The output of the matched filter is not a replica of the input signal. However, from the point of view of detecting signals in noise, preserving the shape of the signal is of no importance. If it is necessary to preserve the shape of the input pulse rather than maximize the output signal-to-noise ratio, some other criterion must be employed.

The output of the matched filter may be shown to be proportional to the input signal cross-correlated with a replica of the transmitted signal, except for the time delay t_1. The cross-correlation function $R(t)$ of two signals $y(\lambda)$ and $s(\lambda)$, each of finite duration, is defined as

$$\mathbf{R(t) = \int_{-\infty}^{\infty} y(\lambda)\, s(\lambda - t)\, d\lambda} \qquad (7.27)$$

The output $y_o(t)$ of a filter with impulse response $h(t)$ when the input is $y_{in}(t) = s(t) + n(t)$ is

$$\mathbf{y_o(t) = \int_{-\infty}^{\infty} yin(\lambda)\, h(\lambda - t)\, d\lambda} \qquad (7.28)$$

If the filter is a matched filter, then $h(\lambda) = s(t_1 - \lambda)$ and Eq. (7.28) becomes

$$\mathbf{y_o(t) = \int_{-\infty}^{\infty} yin(\lambda)\, s(t_1 - t + \lambda)\, d\lambda} \qquad (7.29)$$

Thus the matched filter forms the cross correlation between the received signal corrupted by noise and a replica of the transmitted signal. The replica of the transmitted signal is "built in" to the matched filter via the frequency-response function. If the input signal $y_{in}(t)$ were the same as the signal $s(t)$ for which the matched filter was designed (that is, the noise is assumed negligible), the output would be the autocorrelation function. The autocorrelation function of a rectangular pulse of width τ is a triangle whose base is of width 2τ.

7.5. Cross Correlation Receiver

Equation (7.29) describes the output of the matched filter as the cross correlation between the input signal and a delayed replica of the transmitted signal. This implies that the matched-filter receiver can be replaced by a cross-correlation receiver that performs the same mathematical operation as shown in Fig. 7.2. The input signal y(t) is multiplied by a delayed replica of the transmitted signal $s(t - T_r)$, and the product is passed through a low-pass filter to perform the integration. The cross-correlation receiver of Fig. 7.2 tests for the presence of a target at only a single time delay T_r Targets at other time delays, or ranges, might be found by varying T_r. However, this requires a longer search time. The search time can be reduced by adding parallel channels, each containing a delay line corresponding to a particular value of T_r as
well as a multiplier and low-pass filter. In some applications it may be possible to record the signal on some storage medium, and at a higher playback speed perform the search sequentially with different values of T_r That is, the playback speed is increased in proportion to the number of time-delay intervals T_r, that are to be tested.

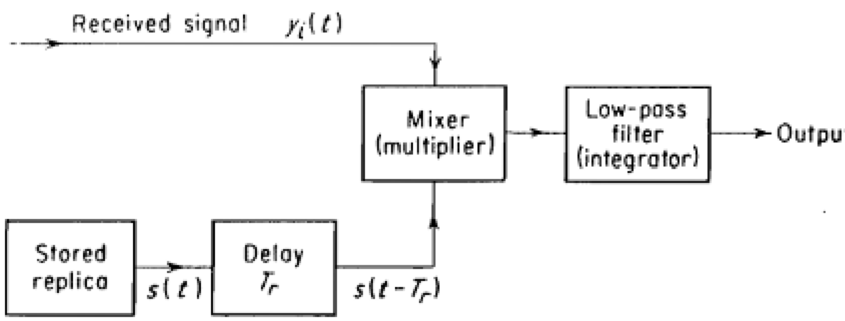

Fig 7.2: Block diagram of a cross-correlation receiver.

Since the cross-correlation receiver and the matched-filter receiver are equivalent mathematically, the choice as to which one to use in a particular radar application is determined by which is more practical to implement. The matched-filter receiver, or an approximation, has been generally preferred in the vast majority of applications.

Essay type questions

1. Write a detailed note on matched filter receiver. [JNTU May 2013]
2. Explain about correlation function and Cross-correlation receiver. [JNTU May 2013]
3. Write the short notes on matched filter. [JNTU May 2012]
4. Write the short notes on non-matched filter. [JNTU May 2012]
5. Explain the principle and characteristics of a matched filter hence derive the expression for frequency response function. [JNTU May 2012] [JNTU May 2011]
6. Explain in detail about Efficiency of non-matched filters compared with the matched filter. [JNTU May 2012]
7. Discuss about efficiency of non-matched filters. [JNTU May 2012]
8. Discuss about Matched filter with nonwhite noise. [JNTU May 2012]
9. Derive the impulse response of a matched filter that is commonly used in a radar receiver. [JNTU May 2011]
10. Discuss about non white noise matched filter. [JNTU May 2011]
11. Discuss about the Fourier transform criterion. [JNTU May 2011]

12. Explain the characteristics of a cross-correlation receiver with a block diagram. [JNTU May 2010]
13. Describe and differentiate between active ECM and passive ECM. [JNTU May 2010]
14. Explain the equivalence between matched filter and correlator? [JNTU Jan 2010]
15. Discuss about the performance of matched filter with non white noise? [JNTU Jan 2010]
16. Draw the block diagram of a correlation receiver and explain its operation with necessary equations. [JNTU Jan 2010]
17. Derive the transfer function for matched filter? [JNTU Jan 2010]
18. Write about the following: [JNTU Jan 2010]
    (i) Coherent detector ii) Likelihood – ratio receiver.
19. What is a matched filter receiver? Draw its response characteristics. [JNTU Jan 2009]
20. Describe the operation of matched filter with non white noise. [JNTU Jan 2009]
21. Derive the matched filter characteristic. [JNTU Jan 2009]
22. Discuss about efficiency of non-matched filters. [JNTU Jan 2009]
23. Derive the equation for impulse response of a matched filter [JNTU Jan 2009]
24. Write short notes on [JNTU Jan 2009]
    i) Efficiency of non matched filters.
    ii) Matched filter with non white noise.

Objective type questions

1. The matched filter may also be specified by

a. $h(t) = \int_{-\infty}^{\infty} H(f)\exp(-j2\pi ft)df$

b. $h(t) = \int_{0}^{\infty} H(f)\exp(-j2\pi ft)df$

c. $h(t) = \int_{-\infty}^{\infty} H(f)\exp(j2\pi ft)df$

d. $h(t) = \int_{0}^{\infty} H(f)\exp(j2\pi ft)df$

2. Peak signal-to-mean noise ratio of the matched filter

a. $R_f = \frac{E}{2N_0}$

b. $R_f \le \frac{2E}{2N_0}$

c. $R_f > \frac{2E}{3N_0}$

d. $R_f \le \frac{2E}{3N_0}$

3. The amplitude spectrum of the matched filter is

a. Obtained by its frequency response

b. Negative of the amplitude spectrum of signal

c. Same as the amplitude spectrum of signal

d. Depends on phase spectrum of signal

4.The noise power per hertz of bandwidth, N0 is equal to

a. $\frac{KT_0}{F}$

b. $\frac{T_0}{2F}$

c. KT_0F

d. $2KT_0F$

5. A network whose frequency response function maximizes the output peak-single-to mean noise ratio is called a

a. Envelope Detector

b. Matched Detector

c. Matched Filter

d. Optimum filter

6. If the band width of the receiver passband is wide compared with that occupied by the signal energy

a. Extraneous noise is introduced

b. Noise is reduced

c. Increases the signal to noise ratio

d. Frequency response is improved

7. If the receiver bandwidth is narrower than the bandwidth occupied by the signal

a. Extraneous noise is introduced b. Noise energy is reduced

c. Increases the signal-to-noise ratio d. Frequency response is improved

8. When there is optimum bandwidth

a. Signal to noise ratio is minimum b. Signal to noise ratio is maximum

c. Noise energy is maximum d. Noise energy is reduced

9. Phase spectrum of the matched filter is

a. Inversely proportional to frequency b. Negative of the phase spectrum of signal

c. Same as the phase spectrum of signal d. Proportional to amplitude

10. The Output of the matched filter is

a. Replica of the input signal b. Proportional to input signal to noise ratio

c. Not a replica of the input signal d. Not preserving the shape of the input signal

11. The output of the matched filter proportional to

a. Input signal

b. Input signal correlated with a replica of transmitted signal

c. Input signal correlated with a replica of transmitted signal except for time delay t 1

d. Input signal cross correlated with a replica of the transmitted signal except for the time delay t1

12. The auto correlation function of a rectangular pulse of width τ is a [12S03]

a. Square whose base is of width T b. Triangular whose base is of width T

c. Triangular whose base is of width 2T d. Square whose base is of width 2T

13. The matched filter forms the

a. Cross correlation between transmitted signals

b. Cross correlation between signal corrupted by noise and replica of transmitted signal

c. Correlation between signal corrupted by noise and replica of transmitted signal

d. Cross correlation between received signal corrupted by noise and a replica of the transmitted signal

14. If the input signal y in(t) were the same as the signal s(t) for which the matched filter was designed, the output would be the

a. Correlation function b. Cross correlation function

c. Auto correlation function d. Replica of the transmitted signal

15. When the input signal s(t) is a rectangular sine wave pulse the output peak-signal to mean noise ratio is

a. Same as signal to noise power ratio

b. Twice the signal to noise power ratio

c. Same as average signal to noise power ratio

d. Twice the average signal to noise power ratio

16. This equation describes the output y0(t) of the matched filter as the

a. Correlation between the input signal and transmitted signal

b. Cross correlation between the input signal and a delayed replica of the transmitted signal

c. Frequency response function   d. Correlation function

17. This requires a longer search time

a. Test for presence of a target at a single time delay

b. Test for presence of targets at time delays founds by varying Tr

c. Test for target at Tr   d. Test for target at (t-Tr)

18. The cross correlation receiver tests for the presence of a target at

a. Only a single time delay   b. Various time delays

c. Two time delays T1 and T 2   d. Two time delays by means of the mixer

19. In cross correlation detection the input signal y(t) is multiplied by a delayed replica of the transmitted signal s(t-Tr) and the product is passed through

a. Mixer to get integrated output   b. Delay Tr

c. Lowpass filter to perform the integration

d. High pass filter to perform the differentiation

20. The matched filter receiver can be replaced by this receiver that performs the same mathematical operation

a. Correlation receiver   b. Cross correlation receiver

c. Super Hetrodyne receiver   d. Non matched filter receiver

21. By adding parallel channels search time can be

a. Increased   b. Reduced   c. Remains constant   d. Zero

22. In correlation detection mixer produces

a. Integrated output of input

b. Product of input and delayed replica of transmitted signal

c. Delayed replica of input signal

d. Delayed replica of transmitted signal

23. In proportion to the number of time delay intervals Tr that are to be tested. The play back speed is

a. Decreased   b. Increased

c. Independent of time delay   d. Twice that of Tr

24. Cross correlation receiver and the matched filter receiver are

a. Not equivalent b. Equivalent practically

c. Equivalent mathematically d. Not equivalent mathematically

25. The frequency response function of the linear time invariant filter

a. Minimizes that output peak signal to noise ratio

b. Maximizes the output peak signal to mean noise ratio

c. Doubles the output peak signal to noise ratio

d. Minimizes the output peak signal to mean noise ratio

26. The maximum efficiency of the single tuned filter occurs for

a. $B\tau = 0.4$ b. $B\tau = 0.6$ c. $B\tau = 1.37$ d. $B\tau = 0.72$

27. For Gaussian pulse input, the maximum efficiency of the Gaussian filter occurs for

a. $B\tau = 0.44$ b. $B\tau = 0.67$ c. $B\tau = 1.37$ d. $B\tau = 0.72$

28. The maximum efficiency of the rectangular filter occurs (when input is rectangular pulse) for

a. $B\tau = 1.37$ b. $B\tau = 0.72$ c. $B\tau = 0.44$ d. $B\tau = 0.67$

29. In Gaussian pulse input rectangular filter loss in SNR compared with matched filter

a. 0.49Db b. 0.85dB c. 0dB d. 0.56dB

30. For rectangular pulse input Gaussian filter have loss in SNR compared with matched filter

a. 0.49 dB b. 0.88 dB c. 0.56 dB d. 0.5 dB

31. The loss in SNR incurred by use of the non matched filter is

a. large b. Small c. Zero d. Same as matched filter

32. If filter consists of S cascaded single tuned stages with rectangular input, loss in SNR compared with matched filter is

a. 0.88 dB b. 0.49 dB c. 0.5 dB d. 0.9 Db

33. The loss in SNR incurred by use of these filters is small

a. Matched filters b. Non matched filters

c. Single tuned filters d. Gaussian filters

34. If filters consisting of 2 cascaded single tuned stages with rectangular pulse input loss in SNR compared with matched filter is

a. 0.88 dB b. 0.49 dB c. 0 dB d. 0.56 dB

35. white noise matched frequency response function reduces to

a. $H(f) = Ga\ S^*(f) \exp(j2\pi ft1)$ b. $H(f) = G\ a\ S^*(f) \exp(-j2\pi ft1)$

c. $H(f) = Ga\ S(f) \exp(-j2\pi ft1)$ d. $H(f) = Ga\ S(f) \exp(j2\pi ft1)$

36. The non white noise matched filter can be considered as the

a. Cascade of whitening and non whitening filters

b. Cascade of whitening and matched filters

c. Cascade of non whitening filters

d. Cascade of matched, non matched filters

37. When NWN matched filter can be considered as cascade of two filters. The first filter is

a. Whitening filter b. Non whitening filter

c. Gaussian filter d. Matched filter

38. When noise is non white, this filter maximizes output signal to noise ratio

a. Whitening filter b. NWN Matched filter

c. Gaussian filter d. Matched filter

39. If the input power spectrum of the interfering noise is given by [N i(f)]2 the frequency response function of the filter which maximizes the output signal to noise ratio is

a. $$H(f) = \frac{G_a S^*(f)\exp(-j2\pi f t_1)}{[N_i(f)]^2}$$

b. $$H(f) = \frac{G_a S^*(f)\exp(j2\pi f t_1)}{N_i^2(f)}$$

c. $$H(f) = \frac{G_a S^*(f)}{N_i^2(f)}$$

d. $$H(f) = \frac{G_a^* S^*(f)\exp(j2\pi f t_1)}{[N_i(f)]^2}$$

40. When the noise is non white, non white noise matched filter

a. Minimizes the output SNR b. Maximizes the output SNR

c. Doubles the output SNR d. Doubles the output SNR divided by F 0

41. For white noise the input power spectrum of the interfering noise is

a. Constant b. Zero c. Increases d. Decreases

42. The whitening filter acts to make

a. Noise spectrum is zero b. Noise spectrum uniform

c. Noise spectrum non uniform d. Finite noise spectrum

43. When NWN matched filter can be considered as cascade of two filters. The second filter is

a. Whitening filter b. Non whitening filter

c. Gaussian filter d. Matched filter

44. If the spectrum of the noise accompanying the signal was assumed to be white it indicates that the matched filter characteristic was

a. Dependent of frequency

b. Independent of frequency

c. Improved

d. Remains constant

45. North filter frequency response function is

a. $H(f) = G_a S^*(f) \exp(-j2\pi f t_1)$

b. $H(f) = G_a \int_{-\infty} S^*(f) \exp(-j2\pi f t_1)$

c. $H(f) = -G_a \int_{-\infty}^{\infty} S^*(f) \exp(j2\pi f t_1)$

d. $H(f) = G_a S^*(f) \exp(j2\pi f t1)$

Answers

Q	A	Q	A	Q	A	Q	A	Q	A	Q	A	Q	A
1	C	2	B	3	C	4	C	5	C	6	A	7	B
8	B	9	B	1	C	11	D	12	C	13	C	14	C
15	D	16	B	17	B	18	A	19	C	20	B	21	B
22	B	23	B	24	C	25	B	26	A	27	A	28	A
29	A	30	A	31	B	32	C	33	B	34	D	35	B
36	B	37	A	38	B	39	A	40	B	41	A	42	B
43	D	44	B	45	A								

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